Exchange Rate Manipulation and Constructive Ambiguity

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Abstract

I explore the implications of central bank transparency during foreign exchange interventions and develop dynamic models in which investors are heterogeneously informed about both interventions and fundamentals. The benchmark two-period model presents the main result that transparency can often exacerbate any misalignment between the exchange rate and fundamentals. This is a consequence of two distinct effects of transparency. First, transparency reveals some information about fundamentals to investors (the truth-telling effect). Second, transparency increases the precision of the exchange rate as a signal of those fundamentals that remain unknown (the signal-precision effect). If a central bank announcement reveals little information about fundamentals, then this second effect dominates and transparency magnifies exchange rate misalignment. In effect, partial information revelation is worse than no information revelation. An important implication of this result is that a policy of ambiguity can increase the effectiveness of intervention to support a declining currency during times of crisis. This matches both central banks' observed behavior in these turbulent episodes and their justifications for more secretive intervention policies.

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1 Introduction

Over the past decade, a growing body of evidence has demonstrated that all but a few countries exert some control over the value of their exchange rates. According to Calvo and Reinhart (2002), this "fear of floating" is common not only among countries that openly admit it, but also among those that claim not to let currency prices affect policy. Just as central banks broadly agree about the desire to control their exchange rates, they broadly disagree about the policies that should accompany these interventions, especially with regard to transparency. In this paper, I develop dynamic models of foreign exchange intervention that address these questions.

I focus on the issue of central bank transparency, specifically on the implications of credible and truthful public announcements about the size and timing of foreign exchange interventions as opposed to deliberate attempts to be secretive and create uncertainty about those interventions. While there are other important aspects of central bank intervention policy, the question of transparency is among both the most important and the most disputed. Indeed, there is extensive evidence that central banks from around the world hold opposing views about the implications of predictability versus unpredictability, and that they implement different policies for different reasons (Bank for International Settlements, 2005; Canales-Kriljenko, 2003; Chiu, 2003).

Two examples from the financial crisis highlight this lack of policy consensus. Both Mexico and Russia faced intense capital outflows and speculative pressure as the price of risky assets throughout the world declined in the months after the collapse of Lehman Brothers in September 2008.[1] The Bank of Mexico has a longtime commitment to transparent foreign exchange intervention, but at the height of this crisis in early February 2009, the Bank became convinced that transparency was hurting its efforts to stabilize the peso and abruptly switched to a secretive and purposely ambiguous policy. In that month alone, the Bank spent nearly two billion dollars of its reserves in unannounced interventions.[2] In this same period, the Bank of Russia fought a protracted battle with the markets over the falling ruble. Its well-publicized attempts to initially guide the currency to an orderly and predictable depreciation eventually gave way to a looser, more ambiguous policy in which the target band for the ruble was substantially widened and made more flexible.[3] Ultimately, the Bank

[1] Between August 2008 and March 2009, both the Mexican peso and the Russian ruble lost more than one third of their values against the US dollar before eventually stabilizing at slightly higher levels.

[2] Although these interventions were intentionally kept secret, the Bank of Mexico did reveal their size afterwards. For a discussion of the Bank's normally transparent policy, see Sidaoui (2005).

[3] In the second half of 2008, the Bank of Russia widened the target band for the ruble to 16.9% (top to bottom) via a series of small adjustments. It then widened the band further to 28.9% in a little over one week in January 2009. Two examples of some of the press coverage surrounding this episode are the articles

of Russia's extensive interventions contributed to a loss of more than 200 billion dollars in foreign exchange reserves (nearly 40% of the Bank's total reserves) in a period of only six months. In both of these cases, policymakers appear to have been uncertain about the best way to complement their interventions and to help effectively stabilize and defend their currencies. In this era of enormous foreign exchange reserves and large-scale interventions, a better understanding of the implications of these different policies is important.

The main prediction of my analysis is that central bank transparency can in fact magnify any existing misalignment between the exchange rate and fundamentals. In all my models, the equilibrium exchange rate is linear and of the form

$$e \quad = \quad \text{fundamentals} \; - \; \text{risk premium} \; + \; \text{misalignment}, \tag{1.1}$$

so exchange rate misalignment refers to the part of the exchange rate other than fundamentals and the risk premium.[4] This paper's main results state that transparency often magnifies the latter misalignment term from equation (1.1). This occurs because a transparent intervention policy improves the precision of the exchange rate as a signal of fundamentals (the signal-precision effect of transparency), and thus compels rational Bayesian investors to weigh that public signal more heavily in their expectations. Although transparency reveals some information about fundamentals (the truth-telling effect of transparency) and thus also diminishes the signal value of the exchange rate, this extra information can be outweighed by the increased precision provided by a public announcement. It is precisely in these cases, when central bank announcements do not credibly reveal sufficient information about fundamentals, that exchange rate misalignment worsens.[5] Figure 1 plots the relationship between exchange rate misalignment and information revelation. As shown, transparency magnifies misalignment for low levels of information revelation but there exists a threshold at which transparency starts to reduce this misalignment. In effect, partial transparency is worse than no transparency, while full transparency is best.

This conclusion has several implications. First, because central banks often intervene in foreign exchange markets to reduce perceived misalignment between the market exchange rate and its long-run equilibrium value (Bank for International Settlements, 2005), my re-

"The Flight from the Rouble" and "Down in the Dumps" from *The Economist*, November 20, 2008 and February 5, 2009, respectively.

[4]This concept of misalignment is closely related to the concept of market depth from the finance and market microstructure literature as discussed by Vives (2008). In particular, more misalignment typically implies less market depth.

[5]In all of the models I present in this paper, the concept of exchange rate misalignment is closely related to the concept of exchange rate informativeness from market microstructure theory. In most cases, more exchange rate misalignment is equivalent to a less informative exchange rate.

sults imply that transparency may sometimes undermine the effectiveness of such interventions. Central banks also intervene to reduce exchange rate volatility (Bank for International Settlements, 2005; Canales-Kriljenko, 2003), and my results imply that transparency may sometimes undermine the effectiveness of such interventions as well. Indeed, I show that exchange rate misalignment as described by equation (1.1) above is an important contributor to exchange rate volatility, so it follows that if transparency can increase misalignment, then transparency can also increase volatility.

Arguably the most important implication of my results, however, is that a policy of ambiguity will often increase the effectiveness of central bank intervention during periods of crisis and large capital outflows. In these episodes, asymmetric information and pro-cyclical liquidity provision often lead to excessive sales of risky assets, as shown by Brunnermeier and Pedersen (2009) and Shleifer and Vishny (1997). My model predicts that it is precisely in situations like these, when risky countries' currencies are undervalued and it is difficult to credibly reveal information about fundamentals, that transparent interventions to support a currency are less effective than more opaque and secretive interventions. In the case of Mexico and Russia, the model argues that both countries would have likely benefited from more secrecy and ambiguity—as they eventually chose—to go along with their extensive foreign exchange interventions.

I build on a simple model of a cashless economy in which investors are heterogeneously informed about both central bank interventions and fundamentals. The main model I present, the benchmark two-period model, posits that foreign exchange interventions contain information about part of exchange rate fundamentals. In the style of Hellwig (1980), information about all future fundamentals is embedded in the current exchange rate so that, by observing the price of currency, investors learn about these fundamentals and update their beliefs. This learning is imperfect, however, as noise traders push the exchange rate away from its fundamental value. Since the price of foreign currency is a publicly observable signal, any time that the exchange rate differs from its fundamental value average beliefs about fundamentals will differ from the true value of fundamentals. Within this framework, I demonstrate that transparency worsens exchange rate misalignment whenever interventions reveal little information about fundamentals.[6]

Throughout this paper, I consider the implications of a policy of publicly and truthfully announcing the size of interventions versus a policy of secrecy. One advantage of focusing on these two policies is that they have a clear economic interpretation in terms of the information sets of investors, making rigorous theoretical analysis easier. In practice, however, a central

[6] In addition to the benchmark model, I include supplemental materials that consider two extensions: an infinite horizon and interventions that respond directly to movements in the exchange rate. The results about transparency are similar to those from the benchmark model in both cases.

bank wishing to be transparent will often announce not only the size of a current intervention, but also the size of past interventions, the size and timing of interventions planned for the future, and the likely stance of other policies in the future.[7] These considerations have a natural interpretation in my models. In particular, all of the results about central bank transparency are statements about the extent of information that is revealed to investors, and the conclusion is that the more information that is credibly communicated through a public announcement, the less likely it is that transparency will exacerbate exchange rate misalignment (as shown in Figure 1).[8]

A truthful central bank announcement affects investors' beliefs in two different ways in my models. First, and more apparently, any parameters the central bank reveals to investors eliminate the role of the exchange rate as a signal of those parameters. This is the *truth-telling effect* of transparency. Second, and less apparently, any parameters the central bank reveals to investors increase the precision of the exchange rate as a signal of other, still-unknown parameters, and hence increase the weight that investors place on the exchange rate signal when forming their beliefs about those unknown parameters. This is the *signal-precision effect* of transparency. These two effects push in opposite directions. The truth-telling effect directly raises expectations of parameters for which average beliefs are too low. This tends to reduce misalignment and appreciate an exchange rate that, because of sales by noise traders, is undervalued relative to fundamentals. Conversely, the signal-precision effect indirectly lowers expectations of parameters for which average beliefs are too low and tends to increase misalignment and further depreciate an already undervalued exchange rate. A large signal-precision effect explains why misalignment increases in the left side of Figure 1 while a large truth-telling effect explains why misalignment decreases in the right side of the figure. The main results of this paper characterize the conditions for which one effect dominates the other.

There are several important conditions that imply that transparency will magnify exchange rate misalignment. The most essential of these is that a central bank announcement reveals only partial information about fundamentals (as shown in Figure 1), a condition that limits the size of the truth-telling effect of transparency relative to the signal-precision effect. If foreign exchange interventions instead contain extensive information about future policies and fundamentals, then a transparent intervention becomes an important and credible source of information, a point emphasized by Dominguez and Frankel (1993a), Mussa (1981), and

[7]Dominguez and Panthaki (2007) and Gnabo et al. (2009) provide empirical evidence that many kinds of central bank statements related to foreign exchange interventions affect the exchange rate.

[8]More precisely, information revelation must surpass some threshold if a public announcement is to reduce exchange rate misalignment, as shown by the non-monotonic relationship in Figure 1.

the whole literature about the signalling hypothesis.[9] My models are consistent with this observation since they predict that transparency reduces exchange rate misalignment and increases the effectiveness of interventions (if the central bank's goal is to reduce misalignment) in these cases. One of this paper's contributions, however, is to build on this logic of the signalling hypothesis by exploring the interaction between partial information revelation and currency mispricing and showing that transparency can in fact exacerbate exchange rate misalignment if interventions are not sufficiently informative about future fundamentals and policies.

The mechanism I describe in this paper matches well with the justification that central banks often provide for their ambiguous policies. In particular, survey evidence from Bank for International Settlements (2005) and Chiu (2003) indicates that central banks worry that unsuccessful transparent interventions might undermine both a bank's credibility and the market's confidence in its currency. Central banks are concerned that highly visible and extensive interventions coupled with continued undesirable movements in the exchange rate will intensify doubts about a bank's ability to achieve its goals. Indeed, a transparent failure of this nature publicly reveals the market's true sentiment about exchange rate fundamentals and magnifies pessimism among market participants with different beliefs. This paper gives these intuitive but vague ideas a precise meaning within a clearly specified economic model.

The paper is organized as follows. Section 2 presents the benchmark two-period model and the main results about central bank transparency. Section 3 concludes. In Section 4.1, there is a brief discussion of the connection between this paper and other related rational expectations asset-pricing models. All proofs are found in Section 4.2. The supplemental materials to this paper present extensions to the benchmark model in which foreign exchange interventions respond directly to exchange rate misalignment and in which there is an infinite horizon.

[9] Sarno and Taylor (2001) and Vitale (2007) both provide excellent surveys of the signalling-hypothesis literature (and the intervention literature, more broadly), while Kaminsky and Lewis (1996) empirically examine the relationship between interventions and future fundamentals.

2 Benchmark Two-Period Model

There are two periods, $t \in \{1, 2\}$, and two countries, home and foreign. I shall refer to the home country's currency as the dollar and the foreign country's currency as the peso. There is only one good and its price in each country is linked by the law of one price, so that $e_t + p_t^* = p_t$ in each period t, where p_t is the log of the price of the good in the home country, p_t^* is the log of the price of the good in the foreign country, and e_t is the log of the nominal exchange rate, which is defined as the dollar price of one peso.

2.1 Assets and Returns

Three assets are traded in this economy: a nominal one-period bond issued by the domestic central bank with return i_1, a nominal one-period bond issued by the foreign central bank with return i_1^*, and a risk-free technology with real return r. The payoffs of all assets are realized in period two. I assume that the domestic central bank credibly commits to a constant domestic price level in all periods so that the interest rate on dollar bonds i_1 is equal to r. Without loss of generality, this constant price level is normalized so that $p_1 = p_2 = 0$, which implies that the log-linearized real return on foreign bonds is equal to $-p_2^* - e_1 + i_1^* = e_2 - e_1 + i_1^*$. For simplicity, I assume that the interest rate in the foreign country in period one i_1^* is also equal to r.

In this benchmark model, the exchange rate in period two is exogenously given by

$$e_2 = f + \kappa, \tag{2.1}$$

where $f \in \mathbb{R}$ represents exchange rate fundamentals in period two and $\kappa \sim \mathrm{N}(0, \sigma_\kappa^2)$ is a shock to the exchange rate in period two.[10] The infinite-horizon extension of this model presented in the supplemental material gives a more precise meaning to the parameters f and κ, as shown by equation (6.16) from Section 6.1. In that model, exchange rate fundamentals are equal to the time-discounted sum of spreads between foreign and domestic interest rates plus the time-discounted sum of risk premia, with the discount factor determined by the structure of the foreign central bank's interest rate rule.[11] The shock to the exchange rate is then the sum of the innovations in the stochastic processes for the foreign central bank's interest rates and purchases of peso bonds.

[10] An alternative interpretation of κ is that it represents the part of fundamentals in period two that cannot be predicted or known in period one. This does not change any of the model's predictions.

[11] Alternatively, in a standard dynamic monetary model, fundamentals are equal to the time-discounted sum of future values of the foreign money supply (relative to the domestic, constant money supply), with the discount factor determined by the semi-elasticity of money demand with respect to the interest rate.

2.2 Foreign Exchange Intervention

The foreign central bank complements its interest rate policy in period one with a foreign exchange intervention in which it purchases $\nu \in \mathbb{R}$ dollars worth of peso bonds. This intervention affects the exchange rate in period one since it changes the total demand for peso bonds in that period.[12] In period two, the relationship between the exchange rate and the central bank's intervention is more complex. I assume that exchange rate fundamentals in period two are given by

$$f = \theta_f f_0 + \theta_\nu f_\nu, \tag{2.2}$$

where f_0 represents the part of fundamentals that is unrelated to the foreign central bank's intervention, f_ν represents the part of fundamentals that is related to the bank's intervention, and $\theta_f, \theta_\nu > 0$ are constants. The constant θ_ν measures the extent of the relationship between fundamentals and the central bank's intervention, with an increase (decrease) in θ_ν corresponding to a greater (lesser) connection between fundamentals and intervention.

To keep this two-period model simple, I assume that the bank's intervention is equal to the part of fundamentals related to that intervention:

$$\nu = f_\nu. \tag{2.3}$$

Equation (2.3) implies that all of the foreign central bank's intervention in period one conveys information about fundamentals and that the central bank knows the true value of f_ν, but it does not necessarily imply that the bank knows the true value of the full exchange rate fundamental f. It is important to emphasize that the model's predictions do not change if this is generalized so that there is a noise term as part of the bank's intervention and hence only a part of this intervention conveys information about fundamentals.[13] This is a particularly relevant generalization. Both Bhattacharya and Weller (1997) and Vitale (1999) start from central bank loss functions and derive optimal intervention rules that consist of one part that is a linear function of fundamentals as in equation (2.3) and another part that is a linear function of the bank's target value for the exchange rate.[14] Because the bank's target is unknown to investors in these authors' setups, this part of the intervention is like

[12] A growing empirical literature emphasizes the immediate and statistically significant effects of foreign exchange interventions on exchange rates. See Ghosh (1992), Dominguez and Frankel (1993b), Ito (2002), Fatum and Hutchison (2003), Payne and Vitale (2003), Chaboud and Humpage (2005), Kearns and Rigobon (2005), and Dominguez and Panthaki (2007), among others.

[13] It is also important to emphasize that the model's basic predictions do not change if the foreign central bank's intervention depends both on fundamentals and on the extent of exchange rate misalignment, as in Popper and Montgomery (2001). This setup is examined in detail in Section 5 of the supplemental materials.

[14] In the context of financial intervention, Bond and Goldstein (2012) also start from a loss function and derive an optimal intervention rule that is linear in fundamentals as in equation (2.3) above.

a noise term.

The relationship between exchange rate fundamentals in period two and the foreign central bank's intervention in period one as described by equations (2.2) and (2.3) merits some discussion. The most narrow interpretation of the constant θ_ν is that it measures only the time-discounted effect of persistent interventions on future risk premia (a determinant of fundamentals), and that interventions are unrelated to all other determinants of the exchange rate. This implies that interventions only have direct, portfolio-balance effects on the exchange rate and are useful as signals about future intervention policy only. In the infinite-horizon extension of this model presented in Section 6 of the supplemental material, I consider precisely this setup. Accordingly, equation (6.16) from that section describes what the parameters θ_f and θ_ν are in a dynamic setting of this kind.

In reality, foreign exchange interventions are likely to convey information about determinants of the exchange rate beyond just future intervention policy. For example, a large foreign exchange intervention may also serve as a highly credible signal of the central bank's future macroeconomic policies (which are a part of exchange rate fundamentals), as emphasized by Dominguez and Frankel (1993a) and Mussa (1981). Even if an intervention is not a clear signal of future policies, it is still likely that the bank's choice of intervention is influenced by its beliefs about fundamentals and its future policy intentions. This point is emphasized by Bhattacharya and Weller (1997) and Vitale (1999), both of who show that in this case an intervention is a function of exchange rate fundamentals and hence represents an important source of information about those fundamentals. In all of these cases, the constant θ_ν will capture more than just the effect of persistent central bank interventions on future risk premia but also any correlation that exists between the foreign central bank's intervention and other exchange rate fundamentals.

2.3 Investors and Information

The economy is populated by a continuum of investors indexed by $i \in [0,1]$. Each investor is endowed with real wealth $w_i > 0$ at the beginning of period one and has negative exponential utility (CARA) over her consumption in period two. Because the log-linearized excess return of peso bonds is equal to $e_2 - e_1 + i_1^* - i_1 = e_2 - e_1$, the maximization problem solved by each investor i is given by

$$\max_{b_i \in \mathbb{R}} \; -E_{i1} \exp\{-\gamma c_{i2}\}, \qquad \text{subject to} \quad c_{i2} = (1+i_1)w_i + (e_2 - e_1)b_i, \tag{2.4}$$

where b_i is the dollar amount of investor i's purchases of peso bonds in period one, c_{i2} is the quantity of the economy's only good consumed by investor i in period two, $\gamma > 0$ is the coefficient of absolute risk aversion, and $E_{i1}[\cdot]$ denotes the conditional expectation with respect to the information set of investor i in period one.

The aggregate demand for peso bonds by the investors is denoted by B. In addition to the investors, the economy is also populated by a mass of noise traders that purchases ξ dollars worth of peso bonds in period one, where $\xi \sim \mathrm{N}(0, \sigma_\xi^2)$. The net supply of peso bonds is equal to zero, so it follows that the market-clearing condition for the peso bond market is given by $B + \nu + \xi = 0$.

The basic setup of the model is common knowledge among all investors. In particular, all investors are aware of the investments available to them and the form of equations (2.1), (2.2), (2.3), and (2.4), and they all observe e_1 publicly in period one. Investors do not publicly observe exchange rate fundamentals, however. Instead, each investor i has uninformative priors for f_0 and ν and receives private signals $x_i = f_0 + \epsilon_i$ and $y_i = \nu + \eta_i$ in period one, where $\epsilon_i \sim \mathrm{N}(0, \sigma_\epsilon^2)$, $\eta_i \sim \mathrm{N}(0, \sigma_\eta^2)$, ϵ_i and η_i are independent, and all noise terms are independent across investors. In equilibrium, investors rationally combine their private signals with the information about both f_0 and ν that is present in the exchange rate in period one. If the foreign central bank chooses to be transparent and publicly announce the value of its intervention ν in period one, then this value becomes common knowledge among all investors.[15] The structure and timing of this two-period benchmark model is summarized in Figure 2.

Because the constant θ_ν measures the extent of the relationship between fundamentals and the foreign central bank's intervention, it also measures the extent of information revelation about fundamentals when the bank publicly and credibly announces the value of ν. In particular, the more information about fundamentals that is contained in the bank's intervention, the more information about fundamentals that is revealed by publicizing that intervention. The central result I present from this two-period model states that information revelation must be large (θ_ν must be large) if transparency is to reduce exchange rate misalignment. This is because the truth-telling effect of transparency is increasing in the extent of information revelation, so that this effect is larger than the signal-precision effect once the information about fundamentals that is revealed by the central bank's intervention is sufficiently extensive.

Let s_i denote the information set of investor i in period one when the foreign central bank makes no announcement about its intervention, and let $\tilde{s}_i$ denote the information set of investor i in period one when the central bank does make such an announcement. It

[15] An alternative interpretation of foreign central bank transparency is that it reduces σ_η and hence reduces the variance of investors' private signals about ν. This alternative interpretation does not change any of the model's basic results about the effects of transparency, as discussed below.

follows, then, that

$$\begin{aligned} s_i &= \{x_i, y_i, e_1\}, \\ \tilde{s}_i &= \{x_i, y_i, e_1, \nu\}, \end{aligned} \tag{2.5}$$

and that $E_{i1}[\cdot]$ is either equal to $E[\cdot \mid s_i]$ or $E[\cdot \mid \tilde{s}_i]$ depending upon the foreign central bank's choice of transparency policy. Similarly, the conditional variance with respect to the information set of investor i in period one is denoted by $\mathrm{Var}_{i1}[\cdot]$.

The aggregate demand for peso bonds by the investors is given by $B = \int_0^1 b_i \, di$, with the understanding that this integral is equal to the average across investors. Similarly, the average expectation of investors in period one is given by $\overline{E}_1[\cdot] = \int_0^1 E_{i1}[\cdot] \, di$, and the average conditional variance of investors in period one is given by $\overline{\mathrm{Var}}_1[\cdot] = \int_0^1 \mathrm{Var}_{i1}[\cdot] \, di$. Finally, let $\sigma^2 = \overline{\mathrm{Var}}_1[e_2]$ denote the average conditional variance of the exchange rate in period two.

2.4 The Equilibrium Exchange Rate

Let $\mathcal{F}$ denote the aggregate state of the economy in period one, so that $\mathcal{F} = \{f_0, \nu, \xi\}$. The equilibrium exchange rate in this setup is a function of this aggregate state, and the goal is to compare the properties of such an equilibrium with and without foreign central bank transparency.

Definition 2.1. An equilibrium of this economy is a function for the exchange rate in period one $e_1 : \mathcal{F} \to \mathbb{R}$ such that: (i) the demand for peso bonds by each investor b_i solves the maximization problem (2.4), where investor i's information set is given by either $s_i = \{x_i, y_i, e_1\}$ if the foreign central bank does not publicly announce the value of ν in period one or $\tilde{s}_i = \{x_i, y_i, e_1, \nu\}$ if the foreign central bank does publicly announce the value of ν in period one; (ii) the peso bond market clears: $B + \nu + \xi = 0$.

Throughout this paper, I restrict the analysis to equilibria in which the exchange rate is a linear function of the aggregate state $\mathcal{F}$. It should be noted that in this definition of equilibrium, the foreign central bank's choice of transparency policy itself does not convey any information about the parameters of the model. All proofs from this section are in Section 4.2.

Proposition 2.2. *If the foreign central bank does not publicly announce the value of ν in period one, then the equilibrium exchange rate is given by*

$$e_1 = f + \gamma\sigma^2\nu + \lambda\xi, \tag{2.6}$$

where λ *and* σ^2 *are such that*

$$\lambda = \frac{\lambda\theta_f^2\sigma_\epsilon^2 + \lambda\theta_\nu(\theta_\nu + \gamma\sigma^2)\sigma_\eta^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + \gamma\sigma^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2} + \gamma\sigma^2, \tag{2.7}$$

$$\sigma^2 = \theta_f^2\sigma_\epsilon^2 + \theta_\nu^2\sigma_\eta^2 + \sigma_\kappa^2 - \frac{\left(\theta_f^2\sigma_\epsilon^2 + \theta_\nu(\theta_\nu + \gamma\sigma^2)\sigma_\eta^2\right)^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + \gamma\sigma^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2}. \tag{2.8}$$

The equilibrium exchange rate as given by equation (2.6) matches with equation (1.1) from the Introduction.[16] The parameter λ in this expression is always positive and measures the magnitude of exchange rate misalignment for any demand by noise traders ξ.[17,18] Indeed, an increase in λ corresponds to an increase in currency mispricing, holding other terms constant. The parameter λ also measures the inverse of market depth as commonly defined in the finance and market microstructure literature (see, for example, Chapter 4 of Vives, 2008). In this literature, an increase in market depth typically lowers exchange rate volatility, a result that is consistent with Proposition 2.2, as I shall discuss below.

A number of important properties of the equilibrium exchange rate from this proposition stand out. First, the effects of noise traders on the exchange rate extend beyond the standard demand channel since $\lambda > \gamma\sigma^2$. In models with rational expectations and heterogeneously informed investors such as this, the equilibrium exchange rate is a publicly observable signal of exchange rate fundamentals f. Noise traders drive the exchange rate away from its fundamental value by altering the total demand for peso bonds, which then biases the average expectations of investors about f. The difference between λ and $\gamma\sigma^2$ captures this extra effect and is exactly equal to the bias in investors' expectations.

A glimpse at the proof of Proposition 2.2 illustrates this point. Market clearing in the peso bond market implies that the exchange rate in period one is of the form

$$e_1 = \overline{E}_1[f] + \gamma\sigma^2(\nu + \xi). \tag{2.9}$$

Solving for the equilibrium requires evaluating the average expectation $\overline{E}_1[f]$ and determining how much weight it places on the noise term ξ. This weight is equal to the bias of

[16]Strictly speaking, the term $-\gamma\sigma^2\nu$ is a measure of the peso bond risk premium that will prevail on average, since the demand by noise traders ξ is equal to zero on average.

[17]All of the numerical solutions to the system of equations (2.7) and (2.8) I have computed indicate that there exists a unique real solution (together with four complex solutions). Even if multiple real solutions do exist for some set of parameters, all of the results about transparency that I present below (Theorem 2.4 and Corollary 2.5) are true for all possible real solutions.

[18]There are several alternative definitions of exchange rate misalignment in this setup, an issue discussed in Section 4.1. I find that these alternative definitions do not meaningfully alter the basic results and implications of the model.

investors' average expectations of fundamentals f which, together with the risk premium term $\gamma\sigma^2$, yields the exchange rate misalignment parameter λ. Evaluating this expectation is accomplished using standard Bayesian formulas, which imply that

$$\overline{E}_1[f] = f + \frac{\mathrm{Cov}_i[f, e_1]}{\mathrm{Var}_i[e_1]}\lambda\xi, \tag{2.10}$$

where $E_i[\cdot], \mathrm{Var}_i[\cdot]$, and $\mathrm{Cov}_i[\cdot,\cdot]$ denote, respectively, the expected value, variance, and covariance with respect to the information set consisting only of the private signals x_i and y_i (no observation of e_1 in this information set). The last term in equation (2.10) is equal to the bias of investors' average expectations of f, and this term reflects the fact that the exchange rate in period one contains information about f (since $\mathrm{Cov}_i[f, e_1]$ is nonzero) and thus its value contributes to equilibrium expectations of this unknown fundamental.

Recall the two distinct effects of transparency: the *truth-telling effect*, which reduces currency mispricing, and the *signal-precision effect*, which magnifies currency mispricing. The truth-telling effect refers to the fact that any parameters the foreign central bank credibly and truthfully reveals to investors eliminate the role of the exchange rate as a signal of those parameters. In this model, if the central bank announces its intervention ν to the public, then investors learn about f_ν and no longer form expectations of this part of exchange rate fundamentals. In terms of equation (2.10), this lowers the value of $\mathrm{Cov}_i[f, e_1]$ and hence reduces exchange rate misalignment, λ. The signal-precision effect refers to the fact that any parameters the central bank reveals to investors also increase the precision of the exchange rate as a signal of other, still-unknown parameters. In this model, if the bank announces its intervention ν to the public, then this increases the precision of the exchange rate as a signal of the part of fundamentals that is not related to this intervention f_0. In terms of equation (2.10), this lowers the value of $\mathrm{Var}_i[e_1]$ and hence magnifies exchange rate misalignment, λ. The main result in this paper, Theorem 2.4 below, characterizes precisely when one of these effects dominates the other.

For most parameterizations of this model, the exchange rate misalignment parameter λ is decreasing in the precision of investors' private signals about fundamentals f_0. This property is also apparent in the similar models of Angeletos and Werning (2006) and Vives (2008), the latter of who shows that market depth—equal to $1/\lambda$ in this benchmark model—is increasing in the precision of investors' private signals. Intuitively, this occurs because better information about fundamentals compels investors to trade more aggressively and hence move the value of the exchange rate closer to its fundamental value.

In order to examine the effects of transparency on the price of the peso, it is necessary to solve for the equilibrium exchange rate when the central bank credibly and publicly announces the value of ν in period one. Let $\tilde{e}_1$ denote the exchange rate in period one in this case of transparency.

Proposition 2.3. *If the foreign central bank credibly and publicly announces the value of ν in period one, then the equilibrium exchange rate is given by*

$$\tilde{e}_1 = f + \gamma\tilde{\sigma}^2\nu + \tilde{\lambda}\xi, \tag{2.11}$$

where $\tilde{\lambda}$ and $\tilde{\sigma}^2$ are such that

$$\tilde{\lambda} = \frac{\tilde{\lambda}\theta_f^2\sigma_\epsilon^2}{\theta_f^2\sigma_\epsilon^2 + \tilde{\lambda}^2\sigma_\xi^2} + \gamma\tilde{\sigma}^2, \tag{2.12}$$

$$\tilde{\sigma}^2 = \theta_f^2\sigma_\epsilon^2 + \sigma_\kappa^2 - \frac{\theta_f^4\sigma_\epsilon^4}{\theta_f^2\sigma_\epsilon^2 + \tilde{\lambda}^2\sigma_\xi^2}. \tag{2.13}$$

In contrast to the system of equations from Proposition 2.2, this system of equations is simple enough to solve analytically and always has a unique real solution. In the equilibrium with transparency, the effects of noise traders on the exchange rate again extend beyond the standard demand channel and bias investors' average expectations of fundamentals. As in the equilibrium with no transparency, the difference between $\tilde{\lambda}$ and $\gamma\tilde{\sigma}^2$ captures this extra effect and is equal to the bias of investors' expectations.

2.5 Implications of Transparency

Just like the case of no transparency, the parameter $\tilde{\lambda}$ from equation (2.11) of Proposition 2.3 is positive and measures the magnitude of misalignment between the exchange rate and fundamentals. It follows that any time $\tilde{\lambda} > \lambda$, transparency magnifies exchange rate misalignment. The final step is to compare the values of the parameters λ and $\tilde{\lambda}$ and examine when this inequality holds.

Theorem 2.4. *There exists a unique threshold $\hat{\theta}_\nu > 0$ such that $\tilde{\lambda} > \lambda$ if and only if $\theta_\nu < \hat{\theta}_\nu$.*

This threshold is given by $\hat{\theta}_\nu = \tilde{\lambda} - \gamma\tilde{\sigma}^2$, and satisfies

$$\lim_{\sigma_\xi \to 0} \hat{\theta}_\nu = \infty, \qquad \lim_{\sigma_\xi \to \infty} \hat{\theta}_\nu = 0,$$

$$\lim_{\sigma_\kappa \to 0} \hat{\theta}_\nu = \frac{\gamma\theta_f^2\sigma_\epsilon^2}{1 + \gamma^2\theta_f^2\sigma_\epsilon^2\sigma_\xi^2}, \qquad \lim_{\sigma_\kappa \to \infty} \hat{\theta}_\nu = 0,$$

$$\lim_{\theta_f \to 0} \hat{\theta}_\nu = 0, \qquad \lim_{\theta_f \to \infty} \hat{\theta}_\nu = \frac{1}{\gamma\sigma_\xi^2},$$

$$\lim_{\gamma \to 0} \hat{\theta}_\nu = 0, \qquad \lim_{\gamma \to \infty} \hat{\theta}_\nu = 0.$$

Theorem 2.4 presents the main result of the benchmark model and all the extensions presented in this paper.[19] The theorem states that exchange rate misalignment is magnified by transparency ($\tilde{\lambda} > \lambda$) whenever the information content of the central bank's intervention is sufficiently limited ($\theta_\nu < \hat{\theta}_\nu$). Given that central banks interveninig in foreign exchange markets are often concerned with perceived misalignment, this result has clear implications for policy.

Theorem 2.4 also provides insight into the relationship between exchange rate volatility and transparency. In this setup, f and ν are constants, so that the volatility of the exchange rate is simply equal to $\lambda^2\sigma_\xi^2$ and is thus increasing in λ. As a consequence, the theorem also implies that exchange rate volatility is magnified by transparency whenever the information content of the central bank's intervention is sufficiently limited. Much like with perceived misalignment, central banks are also concerned with exchange rate volatility, so it follows by Theorem 2.4 that transparency during foreign exchange interventions may sometimes have undesirable effects on volatility as well.

Corollary 2.5. *If $\theta_\nu < \hat{\theta}_\nu$, then there exists a threshold $\hat{\xi} \in \mathbb{R}$ such that $\tilde{e}_1 < e_1$ if and only if $\xi < \hat{\xi}$.*

Theorem 2.4 and Corollary 2.5 together imply that exchange rate undervaluation together with transparency can in fact magnify currency mispricing and reduce the effectiveness of foreign exchange interventions intended to move the exchange rate closer to its fundamental value. According to the corollary, if transparency increases misalignment, then transparency depreciates the exchange rate relative to ambiguity whenever the peso is sufficiently undervalued relative to fundamentals.

The most important policy implications of these results are likely to apply during times of crisis. In these episodes, asymmetric information and pro-cyclical liquidity provision often

[19]Note that any change in the parameter θ_f is equivalent to a corresponding change in the noise term σ_ϵ. To avoid redundancy, then, Theorem 2.4 does not explicitly characterize the behavior of the threshold $\hat{\theta}_\nu$ as σ_ϵ goes to zero or infinity.

lead to excessive sales of risky assets, as shown by Brunnermeier and Pedersen (2009) and Shleifer and Vishny (1997). This translates to a negative value of ξ in this benchmark model, so that if an intervention does not contain much information about future policies and fundamentals ($\theta_\nu < \hat{\theta}_\nu$), Corollary 2.5 implies that a public announcement about that intervention often depreciates the exchange rate. In this case, the central bank can achieve a higher exchange rate if it does not publicly announce the size of its intervention.

Theorem 2.4 implies that it is only if the information revealed by a public announcement of the foreign central bank's intervention is sufficiently incomplete ($\theta_\nu < \hat{\theta}$) that exchange rate misalignment may be magnified by transparency. In terms of the truth-telling and signal-precision effects of transparency, the theorem states that it is precisely when information revelation is incomplete that the truth-telling effect is small relative to the signal-precision effect. If information revelation is complete ($\theta_\nu > \hat{\theta}_\nu$), on the other hand, Theorem 2.4 implies that the truth-telling effect will exceed the signal-precision effect and transparency will lessen exchange rate misalignment. While this analysis ignores the effect that transparency has on the conditional variance of the exchange rate in period two (which is part of the peso bond risk premium $\gamma\sigma^2$), it captures the essence of how transparency affects the equilibrium outcome of the model.

Consider two special cases. First, in the limit as $\theta_\nu \to 0$, the foreign central bank's intervention in period one neither directly affects nor conveys any information about exchange rate fundamentals in period two. This intervention introduces only noise into the exchange rate in period one. In this case, learning the value of ν tells investors nothing about fundamentals f and eliminates none of the bias of investors' expectations of f, but it does increase the precision of e_1 as a signal of f. This means that there is no truth-telling effect and only a signal-precision effect of transparency. Theorem 2.4 confirms that this is indeed the case, since the threshold $\hat{\theta}_\nu$ is always positive and hence $\theta_\nu < \hat{\theta}_\nu$ and $\tilde{\lambda} > \lambda$ once θ_ν is sufficiently close to zero.

Second, in the limit as $\theta_f \to 0$, the foreign central bank's intervention in period one fully reveals all future exchange rate fundamentals (since f_ν becomes all of fundamentals). Much of the early literature about the signalling hypothesis, such as Dominguez and Frankel (1993a) and Mussa (1981), posits an environment similar to this special case when arguing that transparency is desirable and can effectively reduce exchange rate misalignment. Theorem 2.4 demonstrates that this benchmark model is consistent with these authors' analysis, since $\hat{\theta}_\nu \to 0$ as $\theta_f \to 0$ and θ_ν is positive by assumption. It is important to emphasize, however, that as the information about future fundamentals that is embedded in the central bank's intervention declines, the benefits of transparency become more tenuous.

The behavior of λ relative to $\tilde{\lambda}$ is shown graphically in Figure 3. The baseline parameterization of the model shown in this figure is chosen to match the baseline parameterization of the richer dynamic model presented in the supplemental materials. In Figure 3, the threshold $\hat{\theta}_\nu$ is given by the point at which the two lines intersect. For this same parameterization, Figures 4 and 5 show how the threshold $\hat{\theta}_\nu$ varies as σ_ξ and θ_f, respectively, vary. Figure 4 shows that $\hat{\theta}_\nu$ decreases from infinity to zero as the unpredictability of noise traders grows from zero to infinity, as proved in Theorem 2.4. Figure 5 shows that, consistent with the discussion in the previous paragraph, $\hat{\theta}_\nu \to 0$ as $\theta_f \to 0$. The figure also shows that the threshold $\hat{\theta}_\nu$ grows to $1/\gamma\sigma_\xi^2$ as the extent to which exchange rate fundamentals are unrelated to interventions θ_f grows to infinity. Note that this is also proved in Theorem 2.4.

One important implication of Theorem 2.4 is that whether or not transparency magnifies exchange rate misalignment does not depend on the variance of investors' private signals about central bank interventions σ_η. This follows because the threshold $\hat{\theta}_\nu$ is only a function of the exchange rate parameters $\tilde{\lambda}$ and $\tilde{\sigma}^2$, which do not depend on σ_η since they correspond to a central bank policy of transparency (and hence $\sigma_\eta = 0$). One consequence of this is that changes in the precision of investors' private signals of ν cannot swing the balance between the truth-telling and signal-precision effects of transparency. More precisely, if $\lambda > \tilde{\lambda}$ (or $\lambda < \tilde{\lambda}$), then this relationship must hold for all $\sigma_\eta > 0$.

In fact, I find that increases in the variance of investors' private signals about interventions σ_η tend to magnify the difference between the parameters λ and $\tilde{\lambda}$. This implies that λ is increasing in σ_η whenever $\lambda > \tilde{\lambda}$ and decreasing in σ_η whenever $\lambda < \tilde{\lambda}$. In other words, if transparency increases (decreases) exchange rate misalignment, then all forms of transparency increase (decrease) exchange rate misalignment, regardless of whether transparency reduces or eliminates the variance of investors' private signals about interventions. These properties are shown in Figure 6. This finding is significant because it implies that all of this model's predictions about transparency and exchange rate misalignment apply regardless of how foreign central bank transparency is interpreted. Indeed, all of the main results in this section still obtain even if I follow others such as Morris and Shin (2002), Angeletos and Pavan (2007), and Angeletos et al. (2011), and interpret transparency as any reduction of the noise term σ_η instead of a reduction of this noise term all the way to zero.

This benchmark model formalizes the intuitive but vague justifications that central banks often provide for their ambiguous policies. Theorem 2.4 shows that banks are right to worry that unsuccessful transparent interventions might undermine the market's confidence in their currencies, since transparency makes it easier for investors with different beliefs to learn each others' information and hence for pessimism to intensify and spread. In other words, if investors observe a depreciated currency together with an extensive intervention, then they conclude that fundamentals are worse than they previously thought. This reasoning implies that both Mexico and Russia would have likely benefited from more ambiguous intervention policies during the financial crisis, and it provides an explanation for why Mexico and Russia eventually made such a policy switch.

The model provides two key insights that guide this intuition of the central banks. First, it is only if the information that banks reveal to the public is sufficiently partial that transparency can magnify exchange rate misalignment. If central banks can credibly reveal enough information about fundamentals, then transparency is usually stabilizing and will tend to reduce currency misalignment. This highlights the importance of a central bank's ability to reassure markets by making credible public announcements about current and future policies. Second, if transparency does magnify exchange rate misalignment, then ambiguity appreciates only an undervalued currency. This is a direct consequence of rational expectations, and it highlights the importance of information about where the exchange rate is relative to its fundamental value.

Finally, I should emphasize that this model does not imply that an ambiguous intervention policy is always better than a transparent intervention policy. In fact, a transparent intervention policy is often better even if the conditions of Theorem 2.4 hold and $\tilde{\lambda} > \lambda$. This is because central bank policy is an important determinant of currency risk premia and transparency can be an effective way to reduce these risk premia. The purpose of my analysis is to examine and emphasize a mechanism by which transparency can in fact exacerbate exchange rate misalignment, rather than to capture all of the factors that affect exchange rates. While this mechanism is likely to be quite important during times of great uncertainty about policy and fundamentals, it is unlikely to be as important during more normal times.

3 Conclusion

This paper has theoretically examined the implications of central bank transparency during foreign exchange interventions. The central feature of all my models is that investors are heterogeneously informed about both interventions and fundamentals. Information about future fundamentals is embedded in the current exchange rate so that investors learn about these fundamentals when they observe the price of foreign currency.

In this setting, this paper has identified and emphasized two distinct effects of transparency. The first is the truth-telling effect, which corresponds to the fact that any parameters the central bank reveals to investors eliminate the role of the exchange rate as a signal of those parameters. The second is the signal-precision effect, which corresponds to the fact that any parameters the central bank reveals to investors increase the precision of the exchange rate as a signal of other, still-unknown parameters. The truth-telling effect directly raises expectations of parameters for which average beliefs are too low, while the signal-precision effect indirectly lowers expectations of parameters for which average beliefs are too low. I find that the truth-telling effect grows relative to the signal-precision effect as the extent of information about fundamentals that is revealed by a transparent intervention policy increases.

The key implication of my analysis is that central bank transparency can in fact magnify any existing misalignment between the exchange rate and fundamentals. This occurs if a central bank can credibly reveal only partial information about fundamentals to market participants, so that the signal-precision effect of transparency is larger than the truth-telling effect of transparency. In effect, partial information revelation is worse than no information revelation, while full information revelation is best. This result implies that a policy of ambiguity will often increase the effectiveness of central bank intervention during periods of crisis and large capital outflows. In these episodes, asymmetric information and procyclical liquidity provision often lead to excessive sales of risky assets, causing risky countries' currencies to be undervalued and making it difficult to credibly reveal information about fundamentals. This prediction and the intuition behind it match well with the justification that central banks often provide for their ambiguous intervention policies.

Beyond foreign exchange intervention, this paper considers general price manipulation and highlights a mechanism by which transparency can undermine the intended effect of that manipulation. While public information and transparency are normally desirable, I find that if they do not credibly communicate information about fundamentals and future policies, then the signal-precision effect of transparency may lead to undesirable outcomes. Given the ubiquity of price manipulation in foreign exchange markets, these more subtle effects of transparency deserve further analysis and consideration.

4 Appendix

4.1 Discussion: Exchange Rate Misalignment

This paper's main results and implications refer to the effects of transparency on exchange rate misalignment, so it is important to comment about the robustness of those results to alternative definitions of misalignment. According to Proposition 2.2, the equilibrium exchange rate is of the form $e_1 = f + \gamma\sigma^2\nu + \lambda\xi$, where these three terms correspond to fundamentals, the risk premium, and exchange rate misalignment, respectively (see equation (1.1) from Section 1). However, equation (2.9) from Section 2 implies that

$$\overline{E}_1[e_2] - e_1 = \overline{E}_1[f] - e_1 = -\gamma\sigma^2(\nu + \xi), \tag{4.1}$$

so it is reasonable to associate $-\gamma\sigma^2(\nu + \xi)$ with the peso bond risk premium and $(\lambda - \gamma\sigma^2)\xi$ with exchange rate misalignment.

This alternative terminology does not alter the basic results and implications of the benchmark model. Regardless of how exchange rate misalignment is defined, Theorem 2.4 and Corollary 2.5 still imply that transparency will magnify exchange rate volatility and reduce the effectiveness of foreign exchange interventions intended to appreciate an undervalued currency. This latter result is especially important, because it shows that this paper's most important policy implications—which apply to interventions in support of a declining currency during times of crisis—are consistent across all definitions of exchange rate misalignment. Furthermore, all numerical solutions to the benchmark model indicate that the effect of transparency on the quantities $\lambda - \gamma\sigma^2$ and σ^2 is the same as the effect of transparency on λ. To be precise, these solutions indicate that $\tilde{\lambda} - \gamma\tilde{\sigma}^2 > \lambda - \gamma\sigma^2$ if and only if $\tilde{\lambda} > \lambda$, and also $\tilde{\sigma}^2 > \sigma^2$ if and only if $\tilde{\lambda} > \lambda$. If transparency magnifies exchange rate misalignment as measured by the parameter λ, then transparency also magnifies exchange rate misalignment as measured by $\lambda - \gamma\sigma^2$, $\lambda + \gamma\sigma^2$, or σ^2. In other words, the model's basic results about transparency and exchange rate misalignment are the same for all of these different definitions of misalignment.

Another alternative is to compare the equilibrium exchange rate in the benchmark model with the exchange rate that obtains if investors are perfectly informed about fundamentals f. The difference between the exchange rate under these two information structures measures the extent of exchange rate misalignment generated by the imperfect information of investors, which is similar to the misalignment emphasized by Bacchetta and van Wincoop (2006) and Nimark (2012), among others. If investors are perfectly informed about fundamentals f, then average expectations are such that $\overline{E}_1[f] = f$ and $\sigma^2 = \overline{\text{Var}}_1[e_2] = \overline{\text{Var}}_1[f + \kappa] = \sigma_\kappa^2$. According to equation (4.1) above, then, the equilibrium exchange rate both with and without transparency in this case is given by

$$e_1 = \tilde{e}_1 = f + \gamma\sigma_\kappa^2\nu + \gamma\sigma_\kappa^2\xi. \tag{4.2}$$

If exchange rate misalignment is measured as the difference between this exchange rate and the exchange rates from Propositions 2.2 and 2.3, then misalignment without transparency is equal to $\gamma(\sigma^2 - \sigma_\kappa^2)\nu + (\lambda - \gamma\sigma_\kappa^2)\xi$ and misalignment with transparency is equal to $\gamma(\tilde{\sigma}^2 - \sigma_\kappa^2)\nu + (\tilde{\lambda} - \gamma\sigma_\kappa^2)\xi$. As discussed in the previous paragraph, however, this alternative definition of exchange rate misalignment does not change any of the conclusions from the benchmark model, since misalignment as measured by $\lambda + \gamma\sigma^2$ is affected by transparency in exactly the same way as is misalignment as measured by λ.

4.2 Proofs

This section presents the proofs of Propositions 2.2 and 2.3, Theorem 2.4, and Corollary 2.5.

Proof of Proposition 2.2 Suppose that the exchange rate in period two is normally distributed conditional on investor i's information set. Then, the investors' problem (2.4) is a standard CARA-normal maximization problem, and the demand for peso bonds by investor i is given by

$$b_i = \frac{E_{i1}[e_2] - e_1}{\gamma \operatorname{Var}_{i1}[e_2]}. \tag{4.3}$$

Suppose also that $\operatorname{Var}_{i1}[e_2]$ is equal for all $i \in [0,1]$ and hence that $\overline{\operatorname{Var}}_1[e_2] = \operatorname{Var}_{i1}[e_2]$. It follows that $\sigma^2 = \operatorname{Var}_{i1}[e_2]$ and that the aggregate investor demand for peso bonds in period one is given by

$$B = \frac{\overline{E}_1[e_2] - e_1}{\gamma\sigma^2}, \tag{4.4}$$

which, together with the market clearing condition in the peso bond market, implies that

$$e_1 = \overline{E}_1[e_2] + \gamma\sigma^2(\nu + \xi). \tag{4.5}$$

The exchange rate in period two is given by $e_2 = \theta_f f_0 + \theta_\nu f_\nu + \kappa$, so that $E_{i1}[e_2] = \theta_f E_{i1}[f_0] + \theta_\nu E_{i1}[\nu]$ (recall that $f_\nu = \nu$ by equation (2.3)). I am interested in the rational expectations equilibrium of this economy, so investors must take into account the fact that the value of the exchange rate in period one is a signal of both f_0 and ν. In other words, the exchange rate e_1 is part of investors' information sets in period one.

Let $E_i[\cdot]$, $\operatorname{Var}_i[\cdot]$, and $\operatorname{Cov}_i[\cdot,\cdot]$ denote, respectively, the expected value, variance, and covariance with respect to the information set consisting only of the private signals x_i and y_i. In equilibrium, the exchange rate in period one is of the form

$$e_1 = f + \gamma\sigma^2\nu + \lambda\xi = \theta_f f_0 + (\theta_\nu + \gamma\sigma^2)\nu + \lambda\xi, \tag{4.6}$$

so that $\operatorname{Cov}_i[f_0, e_1] = \theta_f\sigma_\epsilon^2$ and $\operatorname{Cov}_i[\nu, e_1] = (\theta_\nu + \gamma\sigma^2)\sigma_\eta^2$. The goal is to solve for the undetermined coefficients λ and σ^2 in equation (4.6). Standard Bayesian inference implies that the exchange rate in period two is normally distributed conditional on investor i's information set (this justifies the assumption of conditional normality) and that

$$\begin{aligned} E_{i1}[f_0] &= E_i[f_0] + \frac{\operatorname{Cov}_i[f_0, e_1]}{\operatorname{Var}_i[e_1]}(e_1 - E_i[e_1]) \\ &= x_i + \frac{\theta_f\sigma_\epsilon^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + \gamma\sigma^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2}\left(e_1 - \theta_f x_i - (\theta_\nu + \gamma\sigma^2)y_i\right), \end{aligned}$$

and

$$\begin{aligned} E_{i1}[\nu] &= E_i[\nu] + \frac{\operatorname{Cov}_i[\nu, e_1]}{\operatorname{Var}_i[e_1]}(e_1 - E_i[e_1]) \\ &= y_i + \frac{(\theta_\nu + \gamma\sigma^2)\sigma_\eta^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + \gamma\sigma^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2}\left(e_1 - \theta_f x_i - (\theta_\nu + \gamma\sigma^2)y_i\right). \end{aligned}$$

Because $\overline{E}_1[f_0] = \int_0^1 E_{i1}[f_0]\,di$ and $\overline{E}_1[\nu] = \int_0^1 E_{i1}[\nu]\,di$, it follows that

$$\begin{aligned}
\overline{E}_1[f_0] &= \int_0^1 x_i\,di + \frac{\theta_f\sigma_\epsilon^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + \gamma\sigma^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2}\left(e_1 - \theta_f \int_0^1 x_i\,di - (\theta_\nu + \gamma\sigma^2)\int_0^1 y_i\,di\right) \\
&= f_0 + \frac{\theta_f\sigma_\epsilon^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + \gamma\sigma^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2}\left(e_1 - \theta_f f_0 - (\theta_\nu + \gamma\sigma^2)\nu\right) \\
&= f_0 + \frac{\lambda\theta_f\sigma_\epsilon^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + \gamma\sigma^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2}\xi, \qquad (4.7)
\end{aligned}$$

and, similarly, that

$$\begin{aligned}
\overline{E}_1[\nu] &= \nu + \frac{(\theta_\nu + \gamma\sigma_1^2)\sigma_\eta^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + \gamma\sigma^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2}\left(e_1 - \theta_f f_0 - (\theta_\nu + \gamma\sigma^2)\nu\right) \\
&= \nu + \frac{\lambda(\theta_\nu + \gamma\sigma_1^2)\sigma_\eta^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + \gamma\sigma^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2}\xi. \qquad (4.8)
\end{aligned}$$

Substituting equations (4.7) and (4.8) into equation (4.5) above yields

$$\begin{aligned}
e_1 &= \theta_f f_0 + (\theta_\nu + \gamma\sigma^2)\nu + \left(\frac{\lambda\theta_f^2\sigma_\epsilon^2 + \lambda\theta_\nu(\theta_\nu + \gamma\sigma^2)\sigma_\eta^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + \gamma\sigma^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2} + \gamma\sigma^2\right)\xi \\
&= f + \gamma\sigma^2\nu + \left(\frac{\lambda\theta_f^2\sigma_\epsilon^2 + \lambda\theta_\nu(\theta_\nu + \gamma\sigma^2)\sigma_\eta^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + \gamma\sigma^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2} + \gamma\sigma^2\right)\xi. \qquad (4.9)
\end{aligned}$$

The next step is to solve for σ^2, the conditional variance of the exchange rate in period two. Because $e_2 = \theta_f f_0 + \theta_\nu\nu + \kappa$, this conditional variance is given by $\sigma^2 = \theta_f^2\overline{\mathrm{Var}}_1[f_0] + \theta_\nu^2\overline{\mathrm{Var}}_1[\nu] + \sigma_\kappa^2 + 2\theta_f\theta_\nu\overline{\mathrm{Cov}}_1[f_0,\nu]$. As before, standard Bayesian inference implies that

$$\begin{aligned}
\overline{\mathrm{Var}}_1[f_0] &= \mathrm{Var}_i[f_0] - \frac{\mathrm{Cov}_i[f_0,e_1]^2}{\mathrm{Var}_i[e_1]} = \sigma_\epsilon^2 - \frac{\theta_f^2\sigma_\epsilon^4}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + \gamma\sigma^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2}, \\
\overline{\mathrm{Var}}_1[\nu] &= \mathrm{Var}_i[\nu] - \frac{\mathrm{Cov}_i[\nu,e_1]^2}{\mathrm{Var}_i[e_1]} = \sigma_\eta^2 - \frac{(\theta_\nu + \gamma\sigma^2)^2\sigma_\eta^4}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + \gamma\sigma^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2},
\end{aligned}$$

and that

$$\overline{\mathrm{Cov}}_1[f_0,\nu] = \mathrm{Cov}_i[f_0,\nu] - \frac{\mathrm{Cov}_i[f_0,e_1]\,\mathrm{Cov}_i[\nu,e_1]}{\mathrm{Var}_i[e_1]} = -\frac{\theta_f(\theta_\nu + \gamma\sigma^2)\sigma_\epsilon^2\sigma_\eta^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + \gamma\sigma^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2}.$$

Substituting the above equations into the expression $\sigma^2 = \theta_f^2\overline{\mathrm{Var}}_1[f_0] + \theta_\nu^2\overline{\mathrm{Var}}_1[\nu] + \sigma_\kappa^2 + 2\theta_f\theta_\nu\overline{\mathrm{Cov}}_1[f_0,\nu]$

then yields

$$\sigma^2 = \theta_f^2\sigma_\epsilon^2 + \theta_\nu^2\sigma_\eta^2 + \sigma_\kappa^2 - \frac{\theta_f^4\sigma_\epsilon^4 + \theta_\nu^2(\theta_\nu+\gamma\sigma^2)^2\sigma_\eta^4 + 2\theta_f^2\theta_\nu(\theta_\nu+\gamma\sigma^2)\sigma_\epsilon^2\sigma_\eta^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu+\gamma\sigma^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2}$$

$$= \theta_f^2\sigma_\epsilon^2 + \theta_\nu^2\sigma_\eta^2 + \sigma_\kappa^2 - \frac{\left(\theta_f^2\sigma_\epsilon^2 + \theta_\nu(\theta_\nu+\gamma\sigma^2)\sigma_\eta^2\right)^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu+\gamma\sigma^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2}. \tag{4.10}$$

Note that this justifies the assumption that the conditional variance is equal for all investors i. The proof of existence is complete once I equate the undetermined coefficients from equation (4.6) above with the implied expressions from equations (4.9) and (4.10).

The system of equations that determines λ and σ^2 jointly is nonlinear and of too high an order to solve analytically. All of the numerical solutions to this system I have computed indicate that there exists a unique real solution (together with four complex solutions). Even if multiple real solutions do exist for some set of parameters, all of the important results about transparency described in Section 2 are true for all possible real solutions. □

Proof of Proposition 2.3 This proof follows the proof of Proposition 2.2 very closely. If I again assume that the exchange rate in period two is normally distributed conditional on investor i's information set, then it can be shown in a similar manner to before that market clearing in the peso bond market implies that $e_1 = \overline{E}_1[e_2] + \gamma\tilde{\sigma}^2(\nu+\xi)$. In equilibrium, this exchange rate is of the form $e_1 = f + \gamma\tilde{\sigma}^2\nu + \tilde{\lambda}\xi$, so that standard Bayesian inference both justifies the assumption of conditional normality and yields aggregate expectations about f_0 that are similar to those when ν remained unknown:

$$\overline{E}_1[f_0] = f_0 + \frac{\tilde{\lambda}\theta_f\sigma_\epsilon^2}{\theta_f^2\sigma_\epsilon^2 + \tilde{\lambda}^2\sigma_\xi^2}\xi. \tag{4.11}$$

Substituting this equation into the expression for the exchange rate in period one yields

$$\tilde{e}_1 = \theta_f f_0 + (\theta_\nu + \gamma\tilde{\sigma}^2)\nu + \left(\frac{\tilde{\lambda}\theta_f^2\sigma_\epsilon^2}{\theta_f^2\sigma_\epsilon^2 + \tilde{\lambda}^2\sigma_\xi^2} + \gamma\tilde{\sigma}^2\right)\xi$$

$$= f + \gamma\tilde{\sigma}^2\nu + \left(\frac{\tilde{\lambda}\theta_f^2\sigma_\epsilon^2}{\theta_f^2\sigma_\epsilon^2 + \tilde{\lambda}^2\sigma_\xi^2} + \gamma\tilde{\sigma}^2\right)\xi. \tag{4.12}$$

The conditional variance of the exchange rate in period two, $\tilde{\sigma}^2$, is also determined in a manner similar to the previous proof. In particular, standard Bayesian inference implies that

$$\overline{\mathrm{Var}}_1[f_0] = \sigma_\epsilon^2 - \frac{\theta_f^2\sigma_\epsilon^4}{\theta_f^2\sigma_\epsilon^2 + \tilde{\lambda}^2\sigma_\xi^2}.$$

The computation is simpler in this case because ν is known with certainty and hence both $\overline{\mathrm{Var}}_1[\nu]$ and $\overline{\mathrm{Cov}}_1[f_0,\nu]$ are equal to zero. It follows, then, that

$$\tilde{\sigma}^2 = \theta_f^2\overline{\mathrm{Var}}_1[f_0] + \sigma_\kappa^2 = \theta_f^2\sigma_\epsilon^2 + \sigma_\kappa^2 - \frac{\theta_f^4\sigma_\epsilon^4}{\theta_f^2\sigma_\epsilon^2 + \tilde{\lambda}^2\sigma_\xi^2}, \tag{4.13}$$

which shows that the conditional variance is equal for all investors i, and together with equation

(4.12) completes the proof of existence. In this simpler case, the system of equations (2.12) and (2.13) can be solved analytically.

There exists only one real solution to this system and this unique real solution corresponds to the unique equilibrium exchange rate $\tilde{e}_1$. To see that this is the case, note that the third-order polynomial that determines the equilibrium value of $\tilde{\lambda}$ is given by

$$\tilde{\lambda}^3\sigma_\xi^2 = \tilde{\lambda}^2\gamma\sigma_\xi^2\left(\theta_f^2\sigma_\epsilon^2 + \sigma_\kappa^2\right) + \gamma\sigma_\kappa^2\theta_f^2\sigma_\epsilon^2.$$

According to Dummit and Foote (2004), the discriminant of this polynomial is equal to

$$-4\gamma^4\sigma_\xi^6\left(\theta_f^2\sigma_\epsilon^2 + \sigma_\kappa^2\right)^3\sigma_\kappa^2\theta_f^2\sigma_\epsilon^2 - 27\gamma^2\sigma_\xi^4\sigma_\kappa^4\theta_f^4\sigma_\epsilon^4,$$

which is strictly less than zero for all valid parameter values. This implies that there is always one unique real root (and two complex conjugate roots) of this polynomial and hence only one unique real equilibrium value for $\tilde{\lambda}$. A similar argument shows that there exists a unique real equilibrium value for $\tilde{\sigma}^2$ as well. □

Proof of Theorem 2.4 I first show that $\lambda > \tilde{\lambda}$ if and only if $\theta_\nu > \lambda - \gamma\sigma^2$, and then show that $\theta_\nu > \tilde{\lambda} - \gamma\tilde{\sigma}^2$ if and only if $\theta_\nu > \lambda - \gamma\sigma^2$. Together, these two facts imply that $\lambda > \tilde{\lambda}$ if and only if $\theta_\nu > \tilde{\lambda} - \gamma\tilde{\sigma}^2$.

According to equation (2.8) from Proposition 2.2,

$$\begin{aligned}
\sigma^2 &= \theta_f^2\sigma_\epsilon^2 + \theta_\nu^2\sigma_\eta^2 + \sigma_\kappa^2 - \frac{\theta_f^4\sigma_\epsilon^4 + 2\theta_f^2\theta_\nu(\theta_\nu+\gamma\sigma^2)\sigma_\epsilon^2\sigma_\eta^2 + \theta_\nu^2(\theta_\nu+\gamma\sigma^2)^2\sigma_\eta^4}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu+\gamma\sigma^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2} \\
&= \sigma_\kappa^2 + \frac{\left((\theta_\nu+\gamma\sigma^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2\right)\theta_f^2\sigma_\epsilon^2 + \left(\theta_f^2\sigma_\epsilon^2 + \lambda^2\sigma_\xi^2\right)\theta_\nu^2\sigma_\eta^2 - 2\theta_f^2\theta_\nu(\theta_\nu+\gamma\sigma^2)\sigma_\epsilon^2\sigma_\eta^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu+\gamma\sigma^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2} \\
&= \sigma_\kappa^2 + \frac{\theta_f^2\gamma^2\sigma^4\sigma_\epsilon^2\sigma_\eta^2 + \lambda^2\theta_f^2\sigma_\epsilon^2\sigma_\xi^2 + \lambda^2\theta_\nu^2\sigma_\eta^2\sigma_\xi^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu+\gamma\sigma^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2},
\end{aligned}$$

so that by equation (2.7) also

$$\lambda = \frac{\lambda\theta_f^2\sigma_\epsilon^2 + \lambda\theta_\nu(\theta_\nu+\gamma\sigma^2)\sigma_\eta^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu+\gamma\sigma^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2} + \gamma\sigma_\kappa^2 + \frac{\gamma(\gamma\sigma^2)^2\theta_f^2\sigma_\epsilon^2\sigma_\eta^2 + \gamma\lambda^2\sigma_\xi^2\theta_f^2\sigma_\epsilon^2 + \gamma\lambda^2\sigma_\xi^2\theta_\nu^2\sigma_\eta^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu+\gamma\sigma^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2}. \tag{4.14}$$

Similarly, equation (2.13) from Proposition 2.3 implies that $\tilde{\sigma}^2 = \sigma_\kappa^2 + \frac{\tilde{\lambda}^2\theta_f^2\sigma_\epsilon^2\sigma_\xi^2}{\theta_f^2\sigma_\epsilon^2+\tilde{\lambda}^2\sigma_\xi^2}$, so that by equation (2.12) also

$$\tilde{\lambda} = \frac{\tilde{\lambda}\theta_f^2\sigma_\epsilon^2 + \tilde{\lambda}^2\gamma\theta_f^2\sigma_\epsilon^2\sigma_\xi^2}{\theta_f^2\sigma_\epsilon^2 + \tilde{\lambda}^2\sigma_\xi^2} + \gamma\sigma_\kappa^2. \tag{4.15}$$

Equations (4.14) and (4.15) imply that

$$\lambda^2\sigma_\xi^2(\lambda - \gamma\theta_f^2\sigma_\epsilon^2 - \gamma\sigma_\kappa^2) = \gamma\sigma_\kappa^2\theta_f^2\sigma_\epsilon^2 + \gamma\sigma_\eta^2\left(\theta_f^2\gamma^2\sigma^4\sigma_\epsilon^2 + \lambda^2\theta_\nu^2\sigma_\xi^2 + \sigma_\kappa^2(\theta_\nu+\gamma\sigma^2)^2 - \lambda\sigma^2(\theta_\nu+\gamma\sigma^2)\right), \tag{4.16}$$

and

$$\tilde{\lambda}^2\sigma_\xi^2(\tilde{\lambda} - \gamma\theta_f^2\sigma_\epsilon^2 - \gamma\sigma_\kappa^2) = \gamma\sigma_\kappa^2\theta_f^2\sigma_\epsilon^2. \tag{4.17}$$

If I let

$$\Delta = \theta_f^2\gamma^2\sigma^4\sigma_\epsilon^2 + \lambda^2\theta_\nu^2\sigma_\xi^2 + \sigma_\kappa^2(\theta_\nu + \gamma\sigma^2)^2 - \lambda\sigma^2(\theta_\nu + \gamma\sigma^2), \tag{4.18}$$

then equations (4.16) and (4.18) imply that

$$\lambda^2\sigma_\xi^2(\lambda - \gamma\theta_f^2\sigma_\epsilon^2 - \gamma\sigma_\kappa^2) = \gamma\sigma_\kappa^2\theta_f^2\sigma_\epsilon^2 + \gamma\sigma_\eta^2\Delta, \tag{4.19}$$

which is equivalent to

$$\lambda = \gamma\theta_f^2\sigma_\epsilon^2 + \gamma\sigma_\kappa^2 + \frac{\gamma\sigma_\kappa^2\theta_f^2\sigma_\epsilon^2}{\lambda^2\sigma_\xi^2} + \frac{\gamma\sigma_\eta^2\Delta}{\lambda^2\sigma_\xi^2}. \tag{4.20}$$

It follows that λ is increasing in Δ with $\lambda = \tilde{\lambda}$ if and only if $\Delta = 0$ or $\sigma_\eta = 0$. Equation (4.20) also implies that if $\sigma_\eta > 0$, then $\lambda > \tilde{\lambda}$ if and only if $\Delta > 0$. The bulk of this proof amounts to showing that $\Delta > 0$ if and only if $\theta_\nu > \lambda - \gamma\sigma^2$.

Before proving these inequalities, note that equation (2.7) implies that

$$\lambda - \gamma\sigma^2 = \frac{\lambda\theta_f^2\sigma_\epsilon^2 + \lambda\theta_\nu(\theta_\nu + \gamma\sigma^2)\sigma_\eta^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + \gamma\sigma^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2},$$

so that

$$(\lambda - \gamma\sigma^2)\left(\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + \gamma\sigma^2)^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2\right) = \lambda\theta_f^2\sigma_\epsilon^2 + \lambda\theta_\nu(\theta_\nu + \gamma\sigma^2)\sigma_\eta^2.$$

Some algebra then yields

$$\begin{aligned}\lambda^2\sigma_\xi^2(\lambda - \gamma\sigma^2) &= \gamma\sigma^2\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + \gamma\sigma^2)\left(\lambda\theta_\nu - (\lambda - \gamma\sigma^2)(\theta_\nu + \gamma\sigma^2)\right)\sigma_\eta^2 \\ &= \gamma\sigma^2\theta_f^2\sigma_\epsilon^2 + \gamma\sigma^2(\theta_\nu + \gamma\sigma^2)(\theta_\nu + \gamma\sigma^2 - \lambda)\sigma_\eta^2,\end{aligned}$$

which, after multiplying both sides by θ_ν, yields

$$\lambda^2\sigma_\xi^2\theta_\nu = \frac{\gamma\sigma^2\theta_\nu}{\lambda - \gamma\sigma^2}\theta_f^2\sigma_\epsilon^2 + \frac{\gamma\sigma^2(\theta_\nu + \gamma\sigma^2)\theta_\nu}{\lambda - \gamma\sigma^2}(\theta_\nu + \gamma\sigma^2 - \lambda)\sigma_\eta^2. \tag{4.21}$$

Equation (4.21) provides two important implications for the proof of Theorem 2.4. First, it follows from the equation that $\lambda^2\sigma_\xi^2\theta_\nu > \gamma\sigma^2\theta_f^2\sigma_\epsilon^2$ if and only if $\theta_\nu > \lambda - \gamma\sigma^2$, since $\frac{\theta_\nu}{\lambda - \gamma\sigma^2} > 1$ if and only if $\theta_\nu > \lambda - \gamma\sigma^2$ if and only if $\theta_\nu + \gamma\sigma^2 - \lambda > 0$. If both sides of equation (4.21) are divided by $\theta_\nu\theta_f^2\sigma_\epsilon^2$, then a similar logic implies that also $\frac{\lambda^2\sigma_\xi^2}{\theta_f^2\sigma_\epsilon^2} > \frac{\gamma\sigma^2}{\lambda - \gamma\sigma^2}$ if and only if $\theta_\nu > \lambda - \gamma\sigma^2$.

Suppose that $\theta_\nu > \lambda - \gamma\sigma^2$, so that $\theta_\nu + \gamma\sigma^2 > \lambda$ as well. As I just showed in equation (4.21), this implies that both $\lambda^2\sigma_\xi^2\theta_\nu > \gamma\sigma^2\theta_f^2\sigma_\epsilon^2$ and $\frac{\lambda - \gamma\sigma^2}{\gamma\sigma^2} > \frac{\theta_f^2\sigma_\epsilon^2}{\lambda^2\sigma_\xi^2}$. It follows that

$$\begin{aligned}(\gamma\sigma^2)^2\theta_f^2\sigma_\epsilon^2 + \theta_\nu^2\lambda^2\sigma_\xi^2 + \sigma_\kappa^2(\theta_\nu + \gamma\sigma^2)^2 &> (\gamma\sigma^2)^2\theta_f^2\sigma_\epsilon^2 + \theta_\nu\gamma\sigma^2\theta_f^2\sigma_\epsilon^2 + \sigma_\kappa^2(\theta_\nu + \gamma\sigma^2)^2 \\ &= \gamma\sigma^2(\theta_\nu + \gamma\sigma^2)\theta_f^2\sigma_\epsilon^2 + \sigma_\kappa^2(\theta_\nu + \gamma\sigma^2)^2 \\ &= (\theta_\nu + \gamma\sigma^2)^2\left(\sigma_\kappa^2 + \theta_f^2\sigma_\epsilon^2\frac{\gamma\sigma^2}{\theta_\nu + \gamma\sigma^2}\right). \end{aligned} \tag{4.22}$$

Suppose now that $\Delta \leq 0$. It follows by equation (4.20), then, that $\lambda \leq \gamma\theta_f^2\sigma_\epsilon^2 + \gamma\sigma_\kappa^2 + \frac{\gamma\sigma_\kappa^2\theta_f^2\sigma_\epsilon^2}{\lambda^2\sigma_\xi^2}$ and hence that

$$\gamma\sigma^2 = \lambda - (\lambda - \gamma\sigma^2) \leq \gamma\theta_f^2\sigma_\epsilon^2 + \gamma\sigma_\kappa^2 + \frac{\gamma\sigma_\kappa^2\theta_f^2\sigma_\epsilon^2}{\lambda^2\sigma_\xi^2} - (\lambda - \gamma\sigma^2). \tag{4.23}$$

Because $\frac{\lambda-\gamma\sigma^2}{\gamma\sigma^2} > \frac{\theta_f^2\sigma_\epsilon^2}{\lambda^2\sigma_\xi^2}$ in this case, inequality (4.23) implies that

$$\gamma\sigma^2 < \gamma\theta_f^2\sigma_\epsilon^2 + \gamma\sigma_\kappa^2 + \frac{(\lambda-\gamma\sigma^2)\gamma\sigma_\kappa^2}{\gamma\sigma^2} - (\lambda - \gamma\sigma^2) = \gamma\theta_f^2\sigma_\epsilon^2 + \gamma\sigma_\kappa^2 + \frac{\lambda-\gamma\sigma^2}{\gamma\sigma^2}(\gamma\sigma_\kappa^2 - \gamma\sigma^2),$$

which then implies that

$$\gamma\sigma^2\left(1 + \frac{\lambda-\gamma\sigma^2}{\gamma\sigma^2}\right) < \gamma\sigma_\kappa^2\left(1 + \frac{\lambda-\gamma\sigma^2}{\gamma\sigma^2}\right) + \gamma\theta_f^2\sigma_\epsilon^2. \tag{4.24}$$

Because $1 + \frac{\lambda-\gamma\sigma^2}{\gamma\sigma^2} = \frac{\lambda}{\gamma\sigma^2}$, inequality (4.24) yields

$$\sigma^2 < \sigma_k^2 + \theta_f^2\sigma_\epsilon^2\frac{\gamma\sigma^2}{\lambda},$$

from which it follows that

$$\begin{aligned}\lambda(\theta_\nu + \gamma\sigma^2)\sigma^2 &< \lambda(\theta_\nu + \gamma\sigma^2)\sigma_k^2 + (\theta_\nu + \gamma\sigma^2)\theta_f^2\sigma_\epsilon^2\gamma\sigma^2 \\ &< (\theta_\nu + \gamma\sigma^2)^2\left(\sigma_\kappa^2 + \theta_f^2\sigma_\epsilon^2\frac{\gamma\sigma^2}{\theta_\nu + \gamma\sigma^2}\right).\end{aligned} \tag{4.25}$$

Note that inequality (4.25) relies on the fact that $\lambda < \theta_\nu + \gamma\sigma^2$ in this case. Of course, inequality (4.25) together with inequality (4.22) from above implies that

$$\lambda(\theta_\nu + \gamma\sigma^2)\sigma^2 < (\gamma\sigma^2)^2\theta_f^2\sigma_\epsilon^2 + \theta_\nu^2\lambda^2\sigma_\xi^2 + \sigma_\kappa^2(\theta_\nu + \gamma\sigma^2)^2,$$

which, because $\Delta = \theta_f^2\gamma^2\sigma^4\sigma_\epsilon^2 + \lambda^2\theta_\nu^2\sigma_\xi^2 + \sigma_\kappa^2(\theta_\nu + \gamma\sigma^2)^2 - \lambda\sigma^2(\theta_\nu + \gamma\sigma^2)$ by equation (4.18), contradicts the assumption that $\Delta \leq 0$ and proves that $\Delta > 0$ if $\theta_\nu > \lambda - \gamma\sigma^2$.

Suppose that $\theta_\nu < \lambda - \gamma\sigma^2$, so that $\theta_\nu + \gamma\sigma^2 < \lambda$. As shown by equation(4.21) and as discussed earlier in the proof, this implies that both $\lambda^2\sigma_\xi^2\theta_\nu < \gamma\sigma^2\theta_f^2\sigma_\epsilon^2$ and $\frac{\lambda-\gamma\sigma^2}{\gamma\sigma^2} < \frac{\theta_f^2\sigma_\epsilon^2}{\lambda^2\sigma_\xi^2}$. It follows that

$$\begin{aligned}(\gamma\sigma^2)^2\theta_f^2\sigma_\epsilon^2 + \theta_\nu^2\lambda^2\sigma_\xi^2 + \sigma_\kappa^2(\theta_\nu + \gamma\sigma^2)^2 &< (\gamma\sigma^2)^2\theta_f^2\sigma_\epsilon^2 + \theta_\nu\gamma\sigma^2\theta_f^2\sigma_\epsilon^2 + \sigma_\kappa^2(\theta_\nu + \gamma\sigma^2)^2 \\ &= \gamma\sigma^2(\theta_\nu + \gamma\sigma^2)\theta_f^2\sigma_\epsilon^2 + \sigma_\kappa^2(\theta_\nu + \gamma\sigma^2)^2 \\ &= (\theta_\nu + \gamma\sigma^2)^2\left(\sigma_\kappa^2 + \theta_f^2\sigma_\epsilon^2\frac{\gamma\sigma^2}{\theta_\nu + \gamma\sigma^2}\right).\end{aligned} \tag{4.26}$$

Suppose now that $\Delta \geq 0$. It follows by equation (4.20), then, that $\lambda \geq \gamma\theta_f^2\sigma_\epsilon^2 + \gamma\sigma_\kappa^2 + \frac{\gamma\sigma_\kappa^2\theta_f^2\sigma_\epsilon^2}{\lambda^2\sigma_\xi^2}$ and hence that

$$\gamma\sigma^2 = \lambda - (\lambda - \gamma\sigma^2) \geq \gamma\theta_f^2\sigma_\epsilon^2 + \gamma\sigma_\kappa^2 + \frac{\gamma\sigma_\kappa^2\theta_f^2\sigma_\epsilon^2}{\lambda^2\sigma_\xi^2} - (\lambda - \gamma\sigma^2). \tag{4.27}$$

Because $\frac{\lambda-\gamma\sigma^2}{\gamma\sigma^2} < \frac{\theta_f^2\sigma_\epsilon^2}{\lambda^2\sigma_\xi^2}$ in this case, inequality (4.27) implies that

$$\gamma\sigma^2 > \gamma\theta_f^2\sigma_\epsilon^2 + \gamma\sigma_\kappa^2 + \frac{(\lambda-\gamma\sigma^2)\gamma\sigma_\kappa^2}{\gamma\sigma^2} - (\lambda-\gamma\sigma^2) = \gamma\theta_f^2\sigma_\epsilon^2 + \gamma\sigma_\kappa^2 + \frac{\lambda-\gamma\sigma^2}{\gamma\sigma^2}(\gamma\sigma_\kappa^2 - \gamma\sigma^2),$$

which then implies that

$$\gamma\sigma^2\left(1+\frac{\lambda-\gamma\sigma^2}{\gamma\sigma^2}\right) > \gamma\sigma_\kappa^2\left(1+\frac{\lambda-\gamma\sigma^2}{\gamma\sigma_1^2}\right) + \gamma\theta_f^2\sigma_\epsilon^2. \tag{4.28}$$

Once again, because $1+\frac{\lambda-\gamma\sigma^2}{\gamma\sigma^2} = \frac{\lambda}{\gamma\sigma^2}$, inequality (4.28) yields

$$\sigma^2 > \sigma_k^2 + \theta_f^2\sigma_\epsilon^2\frac{\gamma\sigma^2}{\lambda},$$

from which it follows that

$$\begin{aligned}\lambda(\theta_\nu+\gamma\sigma^2)\sigma^2 &> \lambda(\theta_\nu+\gamma\sigma^2)\sigma_k^2 + (\theta_\nu+\gamma\sigma^2)\theta_f^2\sigma_\epsilon^2\gamma\sigma^2 \\ &> (\theta_\nu+\gamma\sigma^2)^2\left(\sigma_\kappa^2 + \theta_f^2\sigma_\epsilon^2\frac{\gamma\sigma^2}{\theta_\nu+\gamma\sigma^2}\right). \end{aligned} \tag{4.29}$$

Note that inequality (4.29) relies on the fact that $\lambda > \theta_\nu + \gamma\sigma^2$ in this case. Of course, inequality (4.29) together with inequality (4.26) from above implies that

$$\lambda(\theta_\nu+\gamma\sigma^2)\sigma^2 > (\gamma\sigma^2)^2\theta_f^2\sigma_\epsilon^2 + \theta_\nu^2\lambda^2\sigma_\xi^2 + \sigma_\kappa^2(\theta_\nu+\gamma\sigma^2)^2,$$

which, because $\Delta = \theta_f^2\gamma^2\sigma^4\sigma_\epsilon^2 + \lambda^2\theta_\nu^2\sigma_\xi^2 + \sigma_\kappa^2(\theta_\nu+\gamma\sigma^2)^2 - \lambda\sigma^2(\theta_\nu+\gamma\sigma^2)$, contradicts the assumption that $\Delta \geq 0$ and proves that $\Delta < 0$ if $\theta_\nu < \lambda - \gamma\sigma^2$. This completes the proof that $\Delta > 0$ if and only if $\theta_\nu > \lambda - \gamma\sigma^2$. Note that the argument thus far proves also that if $\sigma_\eta > 0$, then $\Delta = 0$ if and only if $\lambda = \theta_\nu + \gamma\sigma^2$.

It follows, then, that whenever $\sigma_\eta > 0$, $\lambda > \tilde{\lambda}$ if and only if $\theta_\nu > \lambda - \gamma\sigma^2$ and $\lambda = \tilde{\lambda}$ if and only if $\lambda = \theta_\nu + \gamma\sigma^2$. The final step of the proof is to show that $\theta_\nu > \tilde{\lambda} - \gamma\tilde{\sigma}^2$ if and only if $\theta_\nu > \lambda - \gamma\sigma^2$. Suppose that $\tilde{\lambda} - \gamma\tilde{\sigma}^2 \geq \theta_\nu > \lambda - \gamma\sigma^2$. By definition, if $\sigma_\eta = 0$, then $\lambda = \tilde{\lambda}$ and $\sigma^2 = \tilde{\sigma}^2$, so it follows by continuity that there exists some $\sigma_\eta \geq 0$ such that $\theta_\nu = \lambda - \gamma\sigma^2$. By Lemma 4.1, however, this implies that $\lambda = \theta_\nu + \gamma\sigma^2 = \theta_\nu + \gamma\tilde{\sigma}^2 = \tilde{\lambda}$ for all $\sigma_\eta \geq 0$, which contradicts the assumption that $\tilde{\lambda} - \gamma\tilde{\sigma}^2 \geq \theta_\nu > \lambda - \gamma\sigma^2$ for some $\sigma_\eta > 0$. It follows that $\theta_\nu > \tilde{\lambda} - \gamma\tilde{\sigma}^2$. An almost identical argument proves that $\theta_\nu < \tilde{\lambda} - \gamma\tilde{\sigma}^2$ if $\theta_\nu < \lambda - \gamma\sigma^2$. Note that this proof is constructed using the general representation of any equilibrium exchange rate as characterized by Proposition 2.2, so the proof is valid for all possible equilibrium exchange rates of this model. □

Proof of Corollary 2.5 Recall that $e_1 = f + \gamma\sigma^2\nu + \lambda\xi$ and that a similar expression describes $\tilde{e}_1$, with $\tilde{\lambda}$ and $\tilde{\sigma}^2$ replacing λ and σ^2, respectively. It is immediate, then, that $\tilde{e}_1 - e_1$ is strictly increasing in ξ whenever $\tilde{\lambda} > \lambda$ and that for ξ large enough, this quantity is greater than zero regardless of the value of ν. This implies the existence of a unique threshold $\hat{\xi} \in \mathbb{R}$ such that $\tilde{e}_1 < e_1$ if and only if $\xi < \hat{\xi}$. This threshold is decreasing (increasing) in ν whenever $\sigma^2 > \tilde{\sigma}^2$ ($\sigma^2 < \tilde{\sigma}^2$). □

Lemma 4.1. *If $\lambda = \theta_\nu + \gamma\sigma^2$ for some $\sigma_\eta \geq 0$, then $\lambda = \theta_\nu + \gamma\sigma^2 = \theta_\nu + \gamma\tilde{\sigma}^2 = \tilde{\lambda}$ for all $\sigma_\eta \geq 0$.*

Proof. The first step is to show if $\lambda = \theta_\nu + \gamma\sigma^2$, then $\sigma^2 = \tilde{\sigma}^2$ (as well as $\lambda = \tilde{\lambda}$, which was already shown in the proof of Theorem 2.4). Equation (2.13) from Proposition 2.3 implies that

$$(\tilde{\sigma}^2 - \sigma_\kappa^2)(\theta_f^2\sigma_\epsilon^2 + \tilde{\lambda}^2\sigma_\xi^2) = \tilde{\lambda}^2\theta_f^2\sigma_\epsilon^2\sigma_\xi^2. \tag{4.30}$$

Similarly, equation (2.8) from Proposition 2.2 implies that

$$(\sigma^2 - \sigma_\kappa^2)(\theta_f^2\sigma_\epsilon^2 + \lambda^2\sigma_\xi^2) = \lambda^2\theta_f^2\sigma_\epsilon^2\sigma_\xi^2 + \sigma_\eta^2\Delta_\sigma, \tag{4.31}$$

where Δ_σ is given by

$$\Delta_\sigma = (\gamma\sigma^2)^2\theta_f^2\sigma_\epsilon^2 + \lambda^2\theta_\nu^2\sigma_\xi^2 + \sigma_\kappa^2(\theta_\nu + \gamma\sigma^2)^2 - \sigma^2(\theta_\nu + \gamma\sigma^2)^2. \tag{4.32}$$

Together with equation (4.32), equation (4.18) from the proof of Theorem 2.4 implies that

$$\Delta = \Delta_\sigma + \sigma^2(\theta_\nu + \gamma\sigma^2)(\theta_\nu + \gamma\sigma^2 - \lambda).$$

In the proof of Theorem 2.4, I showed that $\Delta = 0$ if and only if $\lambda = \theta_\nu + \gamma\sigma^2$, so it follows that if $\lambda = \theta_\nu + \gamma\sigma^2$ (and hence $\Delta = 0$), then $\Delta_\sigma = 0$ as well. In that proof, I also showed that if $\Delta = 0$, then $\lambda = \tilde{\lambda}$ as well. According to equations (4.30) and (4.31), if $\Delta_\sigma = 0$ and $\lambda = \tilde{\lambda}$, then it must also be true that $\sigma^2 = \tilde{\sigma}^2$.

It follows that if $\lambda = \theta_\nu + \gamma\sigma^2$ for some $\sigma_\eta \geq 0$, then $\lambda = \tilde{\lambda}$, $\sigma^2 = \tilde{\sigma}^2$, and $\Delta = 0$ for this same $\sigma_\eta \geq 0$. According to equations (4.17) and (4.19), any equilibrium values of λ and σ^2 that satisfy equation (4.19) with $\Delta = 0$ must also satisfy equation (4.17). Furthermore, this is true for any value of $\sigma_\eta \geq 0$, since neither side of equation (4.19) depends on σ_η if $\Delta = 0$. Of course, this implies that if $\lambda = \tilde{\lambda}$, $\sigma^2 = \tilde{\sigma}^2$, and, crucially, $\Delta = 0$ for some $\sigma_\eta \geq 0$, then $\lambda = \tilde{\lambda}$ and $\sigma^2 = \tilde{\sigma}^2$ for all $\sigma_\eta \geq 0$. □

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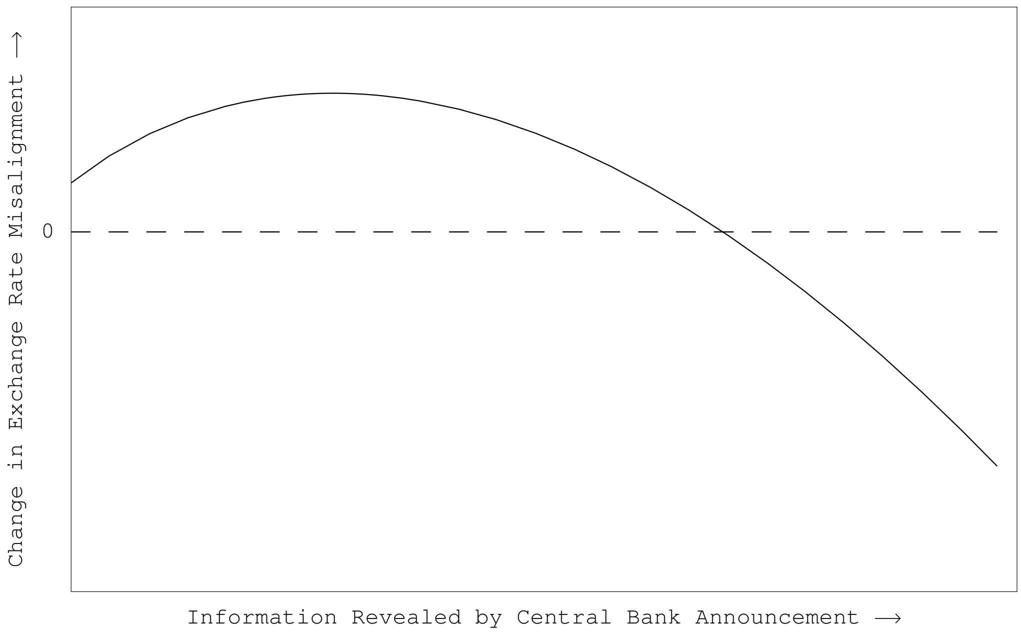

Figure 1: The relationship between exchange rate misalignment and information revelation.

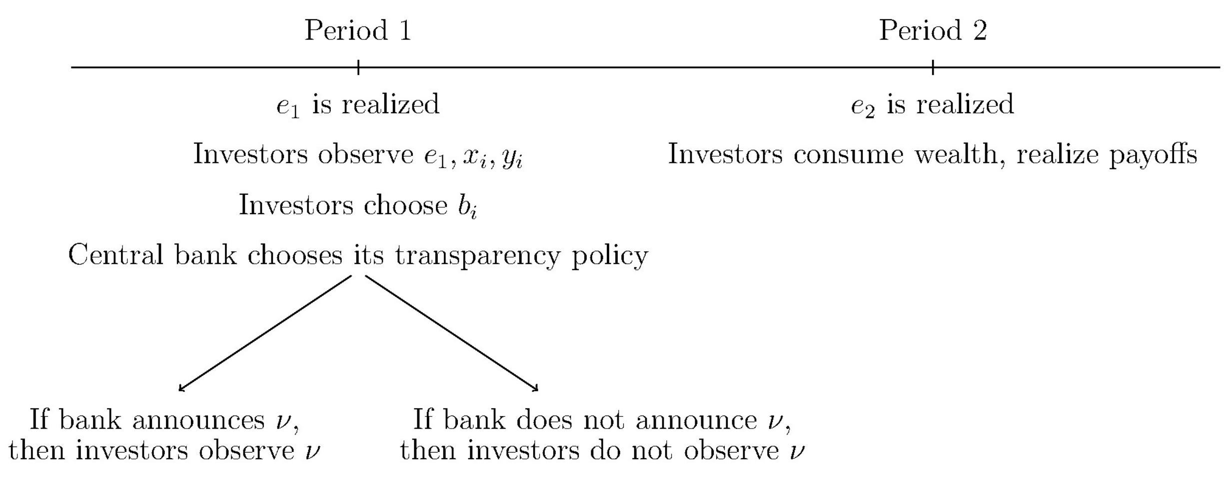

Figure 2: The structure and timing of the two-period benchmark model.

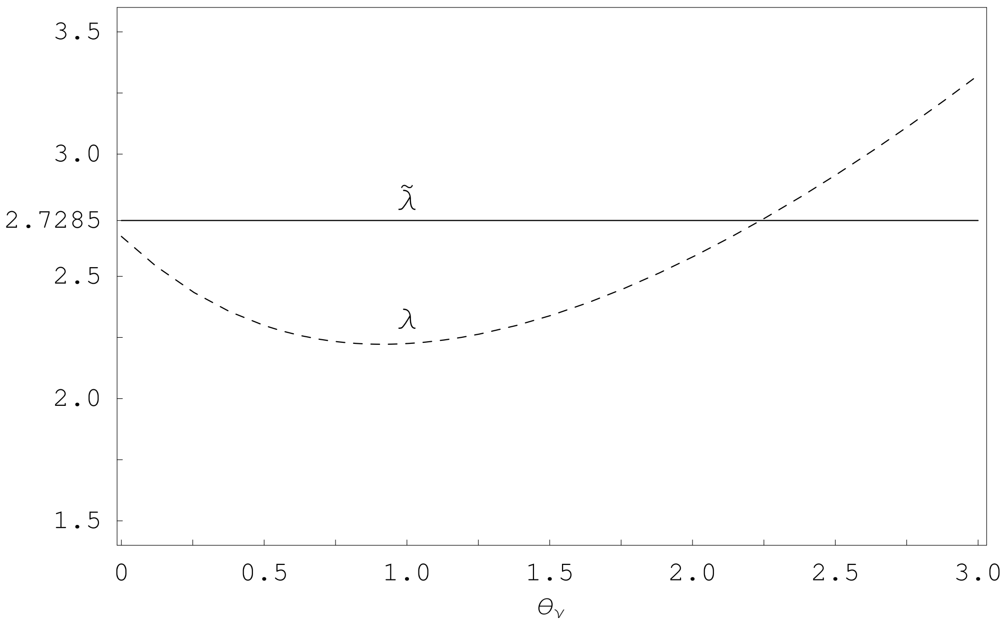

Figure 3: The value of λ (dashed line) and $\tilde{\lambda}$ (solid line) as the level of information revelation θ_ν increases. ($\sigma_\epsilon = 0.35$, $\sigma_\eta = 0.35$, $\sigma_\xi = 0.12$, $\sigma_\kappa = 0.1$, $\gamma = 5$, $\theta_f = 2$)

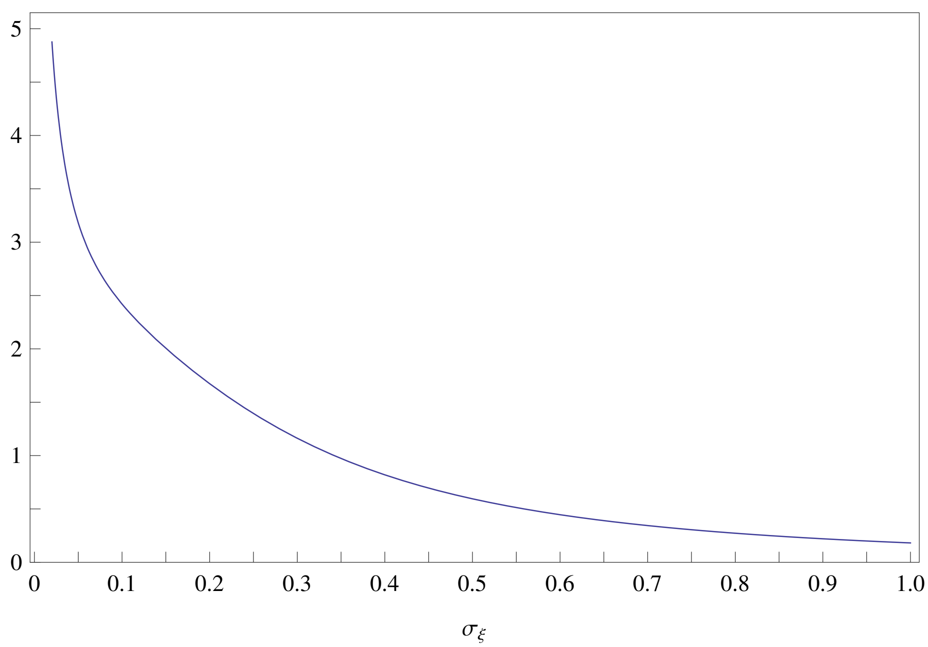

Figure 4: The value of $\hat{\theta}_\nu$ as the variance of noise traders' demand σ_ξ increases. ($\sigma_\epsilon = 0.35, \sigma_\eta = 0.35$, $\sigma_\kappa = 0.1$, $\gamma = 5$, $\theta_f = 2$)

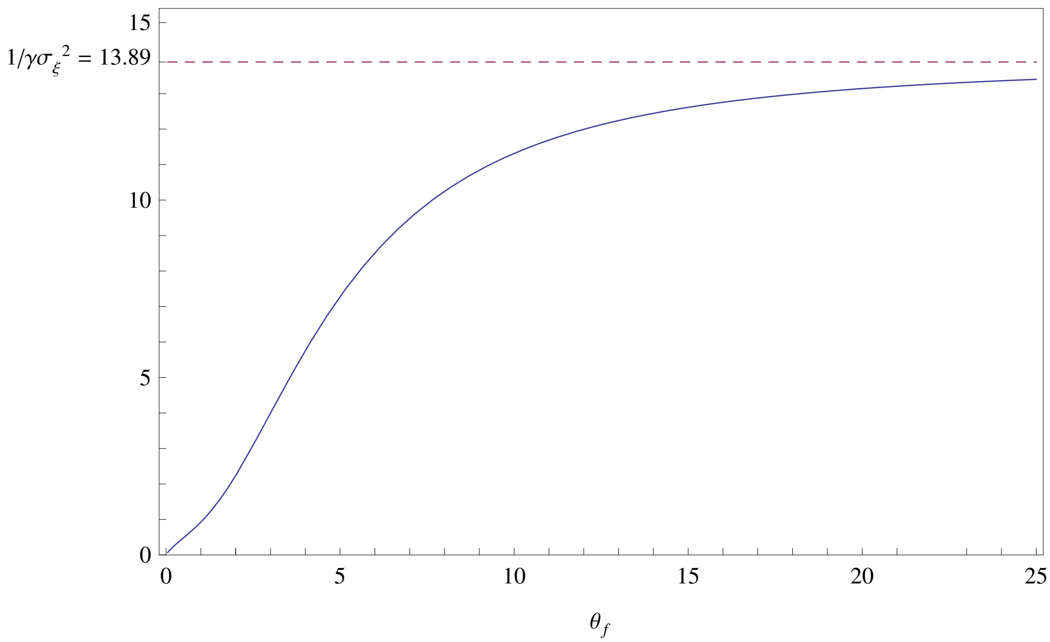

Figure 5: The value of $\hat{\theta}_\nu$ as the extent to which exchange rate fundamentals are unrelated to interventions θ_f increases. ($\sigma_\epsilon = 0.35, \sigma_\eta = 0.35$, $\sigma_\xi = 0.12$, $\sigma_\kappa = 0.1$, $\gamma = 5$)

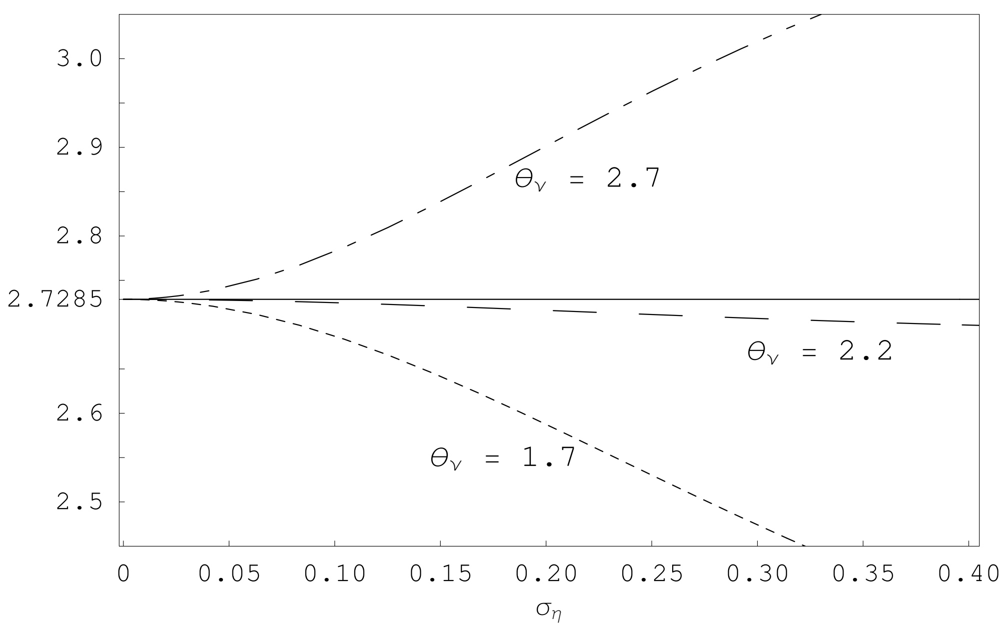

Figure 6: The value of λ as private uncertainty about interventions σ_η increases. ($\sigma_\epsilon = 0.35$, $\sigma_\xi = 0.12$, $\sigma_\kappa = 0.1$, $\gamma = 5$, $\theta_f = 2$)

Exchange Rate Manipulation and Constructive Ambiguity

Supplemental Material

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5 Intervention and Exchange Rate Misalignment

Throughout the benchmark model, I assume that the foreign central bank's intervention is only a function of some part of exchange rate fundamentals. This assumption simplifies the analysis and is sufficient to present this paper's main results and to develop the underlying logic and intuition. In reality, however, a central bank will often take into account more than just its knowledge about fundamentals when choosing how extensively to intervene in the foreign exchange market. An intervening bank is usually also concerned with the value of the exchange rate and the possible presence of misalignment.

There are several reasons why a central bank might intervene in response to movements in the exchange rate. One possibility is that the bank targets some specific value for the exchange rate, as in the intervention models of Bhattacharya and Weller (1997) and Vitale (1999). Another possibility is that the bank's objective is to resist exchange rate misalignment, as in the last section of Chamley (2003). A third possibility is that the central bank wishes to disseminate aggregated private information about exchange rate disturbances generated by noise traders, as in the setup of Popper and Montgomery (2001). Finally, a fourth possibility is that the bank learns about fundamentals from movements in the exchange rate and intervenes based on this learning, as in Bond and Goldstein (2012) and Goldstein et al. (2011). Regardless of the underlying motivation, however, the implication is always that intervention is a function of both exchange rate fundamentals and exchange rate misalignment.[20]

This section extends the benchmark model of Section 2 to include foreign exchange interventions that respond to movements in the exchange rate. I focus primarily on the implications of central bank transparency, with the goal of investigating the robustness of the previous section's results about public announcements and exchange rate misalignment. In this setup, the foreign central bank's intervention ν is both a function of part of exchange rate fundamentals, f_ν, and the noise traders' demand for peso bonds in period one, ξ. As in the previous section, the exchange rate in period two is given by $e_2 = f + \kappa$ (equation 2.1) and exchange rate fundamentals are separated into two parts so that $f = \theta_f f_0 + \theta_\nu f_\nu$ (equation 2.2). In this extended model, however, the foreign central bank's intervention in period one is given by

$$\nu = a_\nu f_\nu + a_\xi \xi, \tag{5.1}$$

where the constants a_ν and a_ξ are such that $a_\nu > 0$ and $-1 < a_\xi < 0$. The assumption that

[20] This assumes that a central bank is not restricted to placing only market orders that cannot depend on the exchange rate, as in the setup of Vitale (1999). Indeed, in order for a foreign exchange intervention to be a function of misalignment, a bank must be able to observe the exchange rate before choosing the size of its intervention.

$a_\nu > 0$ captures the reality that a central bank's choice of foreign exchange intervention is generally positively correlated with some part of exchange rate fundamentals. As described in Section 2, this positive correlation can be the consequence of either interventions that are affected by information about fundamentals (Bhattacharya and Weller, 1997; Vitale, 1999), interventions that are credible signals about future monetary policy (Mussa, 1981), or interventions that permanently alter currency risk premia. The assumption that $-1 < a_\xi < 0$ reflects a focus on interventions that reduce exchange rate misalignment. This is not an important restriction, and the model is easily extended to consider the possibility that $a_\xi > 0$. The form of equation (5.1) is common knowledge among all investors.

As in Section 2, the goal is to examine how a credible and truthful public announcement about ν affects exchange rate misalignment. Because ν is no longer simply equal to f_ν in this section's setup, it is necessary to clarify what private signals the investors observe. In particular, I assume that each investor i receives private signals $x_i = f_0 + \epsilon_i$ and $y_i = f_\nu + \eta_i$ in period one, where $\epsilon_i \sim \mathrm{N}(0, \sigma_\epsilon^2)$, $\eta_i \sim \mathrm{N}(0, \sigma_\eta^2)$, ϵ_i and η_i are independent, and all noise terms are independent across investors. This is equivalent to the benchmark model because in that model ν is equal to f_ν and investors observe private signals about ν (which they know are signals about f_ν, as well). As a consequence, if $a_\nu = 1$, then this section's setup becomes identical to the benchmark setup in the limit as $a_\xi \to 0$, as I demonstrate below.

The definition of an equilibrium exchange rate in this setup is the same as definition 2.1 from Section 2. I also use the same notation, so that e_1 denotes the exchange rate in period one in the absence of a central bank announcement about ν and $\tilde{e}_1$ denotes the exchange rate in period one if there is such an announcement. In addition, this section adopts the previous section's assumptions about investors' preferences and about dollar and peso bonds (so that the log-linearized excess return of peso bonds is equal to $e_2 - e_1$). All proofs from this section are in Section 7.

Proposition 5.1. *If the foreign central bank does not publicly announce the value of ν in period one, then the equilibrium exchange rate is given by*

$$e_1 = f + \gamma\sigma^2\nu + \lambda\xi, \tag{5.2}$$

where λ and σ^2 are given by the solution to

$$\lambda = \frac{\theta_f^2(a_\xi\gamma\sigma^2 + \lambda)\sigma_\epsilon^2 + \theta_\nu(\theta_\nu + a_\nu\gamma\sigma^2)(a_\xi\gamma\sigma^2 + \lambda)\sigma_\eta^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + a_\nu\gamma\sigma_1^2)^2\sigma_\eta^2 + (a_\xi\gamma\sigma^2 + \lambda)^2\sigma_\xi^2} + \gamma\sigma^2, \tag{5.3}$$

$$\sigma^2 = \theta_f^2\sigma_\epsilon^2 + \theta_\nu^2\sigma_\eta^2 + \sigma_\kappa^2 - \frac{\left(\theta_f^2\sigma_\epsilon^2 + \theta_\nu(\theta_\nu + a_\nu\gamma\sigma^2)\sigma_\eta^2\right)^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + a_\nu\gamma\sigma^2)^2\sigma_\eta^2 + (a_\xi\gamma\sigma^2 + \lambda)^2\sigma_\xi^2}. \tag{5.4}$$

A simple comparison of equations (2.7) and (2.8) from Proposition 2.2 with equations (5.3) and (5.4) from Proposition 5.1 shows that if $a_\nu = 1$, then in the limit as $a_\xi \to 0$ the equilibrium exchange rate in this setup converges to the equilibrium exchange rate in the benchmark setup. The parameter λ in the equilibrium exchange rate equation (5.2) is always positive and measures the magnitude of exchange rate misalignment for any demand by noise traders ξ, taking the foreign central bank's intervention ν as given. Because this intervention is a function of both f_ν and ξ (recall from equation 5.1 that $\nu = a_\nu f_\nu + a_\xi \xi$, with $a_\nu > 0$ and $-1 < a_\xi < 0$), the total misalignment of the exchange rate is in fact equal to $(\lambda + a_\xi \gamma \sigma^2)\xi < \lambda\xi$. I focus primarily on the parameter λ rather than $\lambda + a_\xi \gamma \sigma^2$, however, because λ captures the extent of misalignment that exists absent the direct effect of intervention. Indeed, it is obvious that a more extensive intervention more effectively reduces exchange rate misalignment. The more challenging and interesting question is how to maximize the effectiveness of this intervention, holding its size constant. This is answered by examining the parameter λ.

Misalignment-dependent interventions have both direct and indirect effects on exchange rate misalignment. The direct effect refers to the fact that any purchase or sale of peso bonds alters the risk premium and reduces the overall misalignment from $\lambda\xi$ to $(\lambda + a_\xi \gamma \sigma^2)\xi$. The indirect effect refers to the fact that by directly altering misalignment, any purchase or sale of peso bonds also alters the precision of the exchange rate as a signal of fundamentals and hence affects the misalignment that arises from investors' biased expectations. Consider the market clearing condition, which yields an equilibrium exchange rate of the form $e_1 = \overline{E}_1[f] + \gamma\sigma^2(\nu + \xi)$. By equation (5.1), this implies that $e_1 = \overline{E}_1[f] + \gamma\sigma^2 a_\nu f_\nu + \gamma\sigma^2(1 + a_\xi)\xi$. To solve for the misalignment arising from investors' biased expectations, it is necessary to evaluate the average expectation $\overline{E}_1[f]$ and determine how much weight it places on the noise term ξ. This weight tends to decrease as the quantity $\gamma\sigma^2(1 + a_\xi)$ decreases, making investors' expectations less biased. In the limit as $a_\xi \to -1$, the central bank eliminates all of both the bias in investors' expectations and the misalignment in the exchange rate.[21]

Interestingly, however, investors do not learn about fundamentals perfectly in the limit as $a_\xi \to -1$. This is a consequence of the foreign central bank's intervention being a function of the two unknown quantities f_ν and ξ. Indeed, even though $e_1 \to f + \gamma\sigma_1^2 a_\nu f_\nu$, so that the exchange rate is no longer affected by the noise traders' demand for peso bonds, investors still cannot perfectly learn f by observing e_1 since they do not know the value of f_ν. Perhaps surprisingly, this incomplete learning result remains true even if the bank announces its intervention. The next step is to solve for the equilibrium exchange rate in period one if the

[21]Note that the elimination of all misalignment implies that $\lambda + a_\xi \gamma \sigma^2 \to 0$ and hence that λ converges to $\gamma\sigma^2 > 0$ as $a_\xi \to -1$.

foreign central bank truthfully announces the value of ν to the investors.

Proposition 5.2. *If the foreign central bank credibly and publicly announces the value of ν in period one, then the equilibrium exchange rate in period one is given by*

$$\tilde{e}_1 = f + \gamma\tilde{\sigma}^2\nu + \tilde{\lambda}\xi, \tag{5.5}$$

where $\tilde{\lambda}$ and $\tilde{\sigma}^2$ are given by the solution to

$$\tilde{\lambda} = \frac{\tilde{\lambda}a_\nu^2\theta_f^2\sigma_\epsilon^2\sigma_\eta^2}{\theta_f^2(a_\nu^2\sigma_\eta^2 + a_\xi^2\sigma_\xi^2)\sigma_\epsilon^2 + (a_\nu\tilde{\lambda} - a_\xi\theta_\nu)^2\sigma_\eta^2\sigma_\xi^2} + \gamma\tilde{\sigma}^2, \tag{5.6}$$

$$\tilde{\sigma}^2 = \theta_f^2\sigma_\epsilon^2 + \theta_\nu^2\sigma_\eta^2 + \sigma_\kappa^2 - \frac{a_\nu^2\theta_f^2(\theta_f^2\sigma_\epsilon^2 + \theta_\nu^2\sigma_\eta^2)\sigma_\epsilon^2\sigma_\eta^2 + \left(a_\xi\theta_f^2\sigma_\epsilon^2 + \theta_\nu(a_\xi\theta_\nu - a_\nu\tilde{\lambda})\sigma_\eta^2\right)^2\sigma_\xi^2}{\theta_f^2(a_\nu^2\sigma_\eta^2 + a_\xi^2\sigma_\xi^2)\sigma_\epsilon^2 + (a_\nu\tilde{\lambda} - a_\xi\theta_\nu)^2\sigma_\eta^2\sigma_\xi^2}. \tag{5.7}$$

In this equilibrium exchange rate with transparency, the effects of noise traders on the exchange rate again extend beyond the standard demand channel and bias investors' average expectations about fundamentals. As always, the difference between $\tilde{\lambda}$ and $\gamma\tilde{\sigma}^2$ measures the extent of this bias. Equations (5.6) and (5.7) show that if $a_\nu = 1$, then in the limit as $a_\xi \to 0$ the equilibrium exchange rate of Proposition 5.2 converges to the equilibrium exchange rate of Proposition 2.3. The final step is to compare the values of the misalignment parameters λ and $\tilde{\lambda}$.

Theorem 5.3. *There exist thresholds $\hat{\theta}_\nu > 0$, $\widehat{a\sigma} > 0$, $\hat{\sigma}_\eta > 0$, and $\hat{a}_\nu > 0$ such that:*

(i) if $\theta_\nu < \hat{\theta}_\nu$ and $a_\nu\sigma_\eta > \widehat{a\sigma}$, then $\tilde{\lambda} > \lambda$.

(ii) if $\sigma_\eta < \hat{\sigma}_\eta$, then $\tilde{\lambda} < \lambda$.

(iii) if $a_\nu < \hat{a}_\nu$, then $\tilde{\lambda} < \lambda$.

Theorem 5.3 extends the results from Theorem 2.4 in the previous section and presents this section's main results about central bank transparency and exchange rate misalignment. The first part of the theorem states that exchange rate misalignment is magnified by transparency whenever the information content of the central bank's intervention is sufficiently limited ($\theta_\nu < \hat{\theta}_\nu$), and both the investors' private information about the part of fundamentals related to the bank's intervention is sufficiently imprecise and the bank's intervention is sufficiently related to fundamentals ($a_\nu\sigma_\eta > \widehat{a\sigma}$). This result establishes that central bank announcements that reveal little information about fundamentals are likely to exacerbate misalignment, even when those announcements reveal some direct information about that misalignment. Clearly, this conclusion is consistent with the discussion about the potentially undesirable effects of transparency in Section 2.

The second and third parts of the theorem highlight the ways in which the effects of transparency change once the foreign central bank's intervention contains information about the demand of noise traders. In particular, the last two parts of the theorem state that exchange rate misalignment is reduced by transparency whenever either the investors' private information about the part of fundamentals related to the bank's intervention is sufficiently precise ($\sigma_\eta < \hat{\sigma}_\eta$) or the bank's intervention is sufficiently unrelated to fundamentals ($a_\nu < \hat{a}_\nu$). As a consequence, the last parts of Theorem 5.3 imply that a central bank announcement can reduce misalignment even if θ_ν is small and that announcement reveals little direct information about exchange rate fundamentals. Unlike the first part of the theorem, this result contrasts sharply with the previous section's discussion.

The contrasting results of Theorem 5.3 are a direct consequence of the foreign central bank's two-part intervention rule $\nu = a_\nu f_\nu + a_\xi \xi$. Given this rule, an announcement about ν reveals information about both f_ν and ξ without revealing the exact value of either one. As long as this announcement does not reveal precise information about either fundamentals or noise traders' demand for peso bonds, then it is the case that the signal-precision effect of transparency dominates the truth-telling effect as described in the previous section.[22] It is important to emphasize that an announcement that reveals precise information about ξ also reveals precise information about f. The exchange rate in period one is given by $\tilde{e}_1 = f + \gamma\tilde{\sigma}^2\nu + \tilde{\lambda}\xi$, so it follows that if investors learn both ν and ξ from the central bank's announcement, then they can effectively filter all of the noise out of the exchange rate and learn the value of f perfectly. In this case, the truth-telling effect of transparency dominates the signal-precision effect and transparency reduces exchange rate misalignment.

There are three different ways in which an announcement about ν can reveal precise information about f or ξ. One possibility is that fundamentals f are approximately equal to f_ν (because θ_ν is large relative to θ_f), so that information about f_ν is information about nearly all of f and a bank announcement can reduce exchange rate misalignment. This corresponds to the first part of Theorem 5.3, and the logic is the same as it was in Section 2. A second possibility, unique to this section's setup, is that the bank's intervention barely depends on fundamentals and is instead almost entirely a function of ξ. This corresponds to a scenario in which $a_\nu \to 0$ (the third part of Theorem 5.3), and the implication is that an announcement about ν becomes equivalent to an announcement about ξ. A third possibility, also unique to this section's setup, is that investors have very precise private information

[22] Recall that exchange rate fundamentals are given by $f = \theta_f f_0 + \theta_\nu f_\nu$, so information about f_ν is also information about fundamentals.

about f_ν and hence an announcement about ν again becomes equivalent to an announcement about ξ. This corresponds to a scenario in which $\sigma_\eta \to 0$ as in the second part of the theorem.

6 Infinite-Horizon Model

Time is discrete and indexed by t and there are two countries. As in Section 2, I shall refer to the home country's currency as the dollar and the foreign country's currency as the peso. There is only one good for consumption and its price in each country is linked by the law of one price, so that $e_t + p_t^* = p_t$ for all $t \in \mathbb{N}$. As before, the exchange rate is defined as the dollar price of a peso, and its log in period t is given by e_t.

Assets and Returns

In this infinite-horizon extension, three assets are traded in each period: a nominal one-period bond issued by the domestic central bank with return i_t, a nominal one-period bond issued by the foreign central bank with return i_t^*, and a risk-free technology with real return r in each period. As in the two-period model, I assume that the domestic central bank credibly commits to a constant domestic price level in all periods so that the interest rate on dollar bonds i_t is equal to r for all $t \geq 1$. This price level is normalized so that $p_t = 0$, which implies that the log-linearized real return on foreign bonds in period t is equal to $-p_{t+1}^* - e_t + i_t^* = e_{t+1} - e_t + i_t^*$.

The foreign central bank's interest rate policy is more complicated in this setup. In particular, I assume that the foreign central bank follows a Wicksellian interest rate rule in which the price target is equal to zero.[23] This policy is subject to uncertainty, however, so that investors face risk when investing in peso bonds. Specifically, in each period t, the interest rate on peso bonds is given by $i_t^* = ap_t^* + f_t + r$, where f_t follows an autoregressive process of order one (AR(1)) and $a > 0$ is a constant that measures the response of interest rate policy to deviations from the price target. The stochastic process for interest rate deviations is given by $f_t = \rho_f f_{t-1} + \zeta_t$, where $0 < \rho_f < 1$ is a constant and ζ_t is i.i.d. normal, with mean zero and variance σ_ζ^2.

[23] Woodford (2003) provides a detailed discussion of the implications of Wicksellian, price-targeting interest rate rules in cashless economies such as this one.

Foreign Exchange Intervention

As in the two-period model, the foreign central bank complements its interest rate policy by performing foreign exchange interventions in each period. I assume specifically that the central bank purchases $\nu_t \in \mathbb{R}$ dollars worth of peso bonds in each period t and that these interventions follow an AR(1) process, so that $\nu_t = \rho_\nu \nu_{t-1} + \delta_t$, where $0 < \rho_\nu < 1$ is a constant and δ_t is i.i.d. normal, with mean zero and variance σ_δ^2.

This assumption implies that foreign exchange interventions affect exchange rate fundamentals only through their direct effects in this infinite-horizon model. Since the empirical evidence about these direct effects is inconclusive (especially over longer time horizons), I emphasize that this assumption is made only for expositional convenience and that it can be easily relaxed so that interventions also convey information about other exchange rate fundamentals. Indeed, none of this section's qualitative results changes if I assume that interventions are correlated with future interest rates.[24]

Investors and Information

The economy is populated by overlapping generations of investors such that, in each period t, a new generation of investors is born while the old generation of investors dies.[25] Each newly born investor in period t chooses her portfolio and then, in period $t+1$, liquidates her positions and consumes all of her realized wealth before dying. As in the previous section, investors are indexed by $i \in [0,1]$ and each investor i born in period t solves the maximization problem

$$\max_{b_{it} \in \mathbb{R}} -E_{it} \exp\{-\gamma c_{it+1}\}, \qquad \text{subject to} \quad c_{it+1} = (1+i_t)w_{it} + (e_{t+1} - e_t + i_t^* - i_t)b_{it}, \tag{6.1}$$

where $w_{it} > 0$ is investor i's endowment of real wealth at birth, $e_{t+1} - e_t + i_t^* - i_t$ is the log-linearized excess return of peso bonds in period t, b_{it} is the dollar amount of investor i's purchases of peso bonds in period t, c_{it+1} is the quantity of the economy's only good consumed by investor i in period $t+1$, $\gamma > 0$ is the coefficient of absolute risk aversion, and $E_{it}[\cdot]$ denotes the conditional expectation with respect to the information set of investor i in period t.

The aggregate demand for peso bonds by the investors in period t is denoted by B_t. In

[24] Suppose, for example, that the interest rate parameter f_{t+1} is split so that $f_{t+1} = f_{t+1}^0 + \theta_\nu f_{t+1}^\nu$, where $\nu_t = f_{t+1}^\nu$ and $\theta_\nu > 0$. In this case, foreign exchange interventions convey information about future interest rates (exchange rate fundamentals) while all predictions of the model remain the same, except that increases in θ_ν now have the same effect as increases in ρ_ν.

[25] An alternative assumption is that investors live forever and have log preferences, with the risk-free interest rate then determined by the investors' patience. The difficulty with such a setup is that the model becomes intractable once higher-order expectations become part of the equilibrium as in Section 6.2.

addition to the investors, the economy is also populated by a mass of noise traders that purchases ξ_t dollars worth of peso bonds in each period t, where ξ_t is i.i.d. normal, with mean zero and variance σ_ξ^2.[26] Noise traders liquidate all their assets from the previous period before making any purchases. The net supply of peso bonds is constant and equal to zero, so it follows that the market-clearing condition for the peso bond market in each period t is given by $B_t + \nu_t + \xi_t = 0$.

The basic setup of this model is common knowledge among all investors. In particular, all investors are aware of the investments available to them and the form of the stochastic processes for f_t and ν_t, as well as the value of f_t in period t since all current and past interest rates are publicly observable. In this infinite-horizon model, I assume that in each period t each investor i receives the private signals $x_{it} = f_{t+1} + \epsilon_{it}$ and $y_{it} = \nu_t + \eta_{it}$, where $\epsilon_{it} \sim \mathrm{N}(0, \sigma_\epsilon^2)$, $\eta_{it} \sim \mathrm{N}(0, \sigma_\eta^2)$, ϵ_{it} and η_{it} are both i.i.d. and independent of each other, and all noise terms are independent across investors. Following Bacchetta and van Wincoop (2006), I also assume that the generation of investors that is born in period t inherits all of the private information from the generation that dies in period t. More precisely, I assume that in each period t, each newly born investor i inherits all of the private information of investor i from the generation born in period $t-1$.

I shall consider two different specifications for the investors' information. In the first, investors perfectly learn about past values of ν_t which causes higher-order expectations to collapse into more simple average beliefs.[27] The exchange rate can be characterized analytically in this setup, and the equilibrium is similar to the equilibrium from the two-period model in Section 2. It is not surprising, then, that most of the previous conclusions about transparency and exchange rate misalignment continue to be valid. In the second specification, investors do not learn about past values of ν_t so that higher-order expectations remain part of the equilibrium exchange rate. This, however, makes an analytic solution intractable as discussed by Bacchetta and van Wincoop (2006) and Lorenzoni (2009). As a consequence, I solve numerically for an approximate steady-state solution using results from Nimark (2011). Before specifying the details of investors' information sets, it is useful to first solve for the equilibrium exchange rate without any assumptions about these information sets.

In this infinite-horizon setup, I adopt notation similar to that from the benchmark model in the previous section. Let s_{it} denote the information set of investor i in period t when the foreign central bank makes no announcement about its intervention, and let $\tilde{s}_{it}$ denote the information set of investor i in period t when the central bank does make such an

[26] The assumption that noise traders' demand is i.i.d. is made for analytical convenience. The principal results do not change if the model is extended so that shocks to this demand persist over time.

[27] Investors already learn about current and past values of f_t because interest rates are publicly observable.

announcement. Because each newly born investor i inherits all of the private information of investor i from the generation born in period $t-1$, it follows that s_{it} includes s_{it-1} and $\tilde{s}_{it}$ includes $\tilde{s}_{it-1}$. Note that $E_{it}[\cdot]$ is either equal to $E[\cdot \mid s_{it}]$ or $E[\cdot \mid \tilde{s}_{it}]$ depending upon the foreign central bank's choice of transparency policy. Similarly, the conditional variance with respect to the information set of investor i in period t is denoted by $\mathrm{Var}_{it}[\cdot]$.

The aggregate demand for peso bonds by the investors in period t is given by $B_t = \int_0^1 b_{it}\, di$, with the understanding that this integral is equal to the average across investors. Similarly, the average expectation of investors in period t is given by $\overline{E}_t[\cdot] = \int_0^1 E_{it}[\cdot]\, di$, and the average conditional variance of investors in period t is given by $\overline{\mathrm{Var}}_t[\cdot] = \int_0^1 \mathrm{Var}_{it}[\cdot]\, di$. Finally, let $\sigma_t^2 = \overline{\mathrm{Var}}_t[e_{t+1}]$ denote the average conditional variance of the exchange rate in period $t+1$.

The Equilibrium Exchange Rate

Let $\mathcal{F}_0 = \{f_1\}$, and for all $t \in \mathbb{N}$, let $\mathcal{F}_t$ denote the aggregate state of the economy in period t, so that $\mathcal{F}_t = \mathcal{F}_{t-1} \cup \{f_{t+1}, \nu_t, \xi_t\}$.[28] The equilibrium exchange rate in this setup is a function of this aggregate state, and as in Section 2, the goal is to compare the properties of such an equilibrium with and without foreign central bank transparency.

Definition 6.1. A steady-state equilibrium of this economy is a stochastic process for the exchange rate $e_t : \mathcal{F}_t \to \mathbb{R}$, such that for all $t \in \mathbb{N}$: (i) the demand for peso bonds by each investor i solves the maximization problem (6.1), where investor i's information set in period t is given by either s_{it} if the foreign central bank does not publicly announce the value of ν_t in period t or $\tilde{s}_{it}$ if the foreign central bank does publicly announce the value of ν_t in period t; (ii) the peso bond market clears: $B_t + \nu_t + \xi_t = 0$; (iii) the exchange rate is in a steady state: there exists $\sigma^2 > 0$ such that $\sigma_t^2 = \sigma^2$ in all periods $t \in \mathbb{N}$.

Lemma 6.2. *Suppose that the conditional variance* $\mathrm{Var}_{it}[e_{t+1}]$ *is equal for all investors* $i \in [0,1]$ *in all periods* t *and that* e_{t+1} *is normally distributed conditional on the information set of investor* i *in period* t*. Then, a steady-state equilibrium exchange rate satisfies*

$$e_t = \sum_{n=0}^{\infty} \alpha^{n+1} \overline{E}_t^n[f_{t+n}] + \gamma\sigma^2 \sum_{n=0}^{\infty} \alpha^{n+1} \overline{E}_t^n[\nu_{t+n}] + \alpha\gamma\sigma^2\xi_t. \tag{6.2}$$

Proof. If e_{t+1} is normally distributed conditional on the information set of investor i in period t, then problem (6.1) is a standard CARA-normal maximization and the demand for peso

[28] In this setup, investors observe signals of f_{t+1} in period t, so that in some sense (if the probability space and the corresponding filtration were explicitly defined) this interest rate parameter is measurable with respect to time t.

bonds by investor i in period t is given by

$$b_{it} = \frac{E_{it}[e_{t+1}] - e_t + i_t^* - i_t}{\gamma \operatorname{Var}_{it}[e_{t+1}]}. \tag{6.3}$$

If the conditional variance $\operatorname{Var}_{it}[e_{t+1}]$ is equal for all investors $i \in [0,1]$, then $\operatorname{Var}_{it}[e_{t+1}] = \overline{\operatorname{Var}}_t[e_{t+1}] = \sigma_t^2$ and hence

$$B_t = \frac{\overline{E}_t[e_{t+1}] - e_t + i_t^* - i_t}{\gamma \sigma_t^2}. \tag{6.4}$$

Recall that in each period t, the total demand for peso bonds is equal to $B_t + \nu_t + \xi_t$ while the domestic and foreign interest rates are equal to r and $-ae_t + f_t + r$, respectively. In a steady-state equilibrium, $\sigma_t^2 = \sigma^2$ for all t, so that

$$B_t = \frac{\overline{E}_t[e_{t+1}] - (1+a)e_t + f_t}{\gamma \sigma^2}, \tag{6.5}$$

and then, by market clearing,

$$e_t = \alpha \overline{E}_t[e_{t+1}] + \alpha f_t + \alpha\gamma\sigma^2(\nu_t + \xi_t). \tag{6.6}$$

The noise traders' demand is i.i.d. over time, so it follows that $\overline{E}_t[\xi_{t+n}] = 0$ for all $n \geq 1$. Forward iteration of equation (6.6), then, yields

$$e_t = \alpha^2 \overline{E}_t \overline{E}_{t+1}[e_{t+2}] + \alpha^2 \overline{E}_t[f_{t+1}] + \alpha f_t + \alpha^2\gamma\sigma^2 \overline{E}_t[\nu_{t+1}] + \alpha\gamma\sigma^2\nu_t + \alpha\gamma\sigma^2\xi_t \tag{6.7}$$

$$= \alpha^3 \overline{E}_t^3[e_{t+3}] + \sum_{n=0}^{2} \alpha^{n+1} \overline{E}_t^n[f_{t+n}] + \gamma\sigma^2 \sum_{n=0}^{2} \alpha^{n+1} \overline{E}_t[\nu_{t+n}] + \alpha\gamma\sigma^2\xi_t. \tag{6.8}$$

Finally, as demonstrated above, repeated forward iteration implies that the equilibrium exchange rate in period t must satisfy

$$e_t = \sum_{n=0}^{\infty} \alpha^{n+1} \overline{E}_t^n[f_{t+n}] + \gamma\sigma^2 \sum_{n=0}^{\infty} \alpha^{n+1} \overline{E}_t^n[\nu_{t+n}] + \alpha\gamma\sigma^2\xi_t, \tag{6.9}$$

which completes the proof. □

In order to keep the analysis tractable in this infinite-horizon model, I focus only on steady-state equilibria in which the foreign central bank either announces the size of its intervention ν_t in each period t or never announces its intervention. In reality, however, central banks switch between these two policies so that the true steady-state equilibrium is somewhere in between these two extremes. If investors have common knowledge of the past, then the implication of this is only that the true steady-state variances and risk premia with and without transparency are much closer together (depending on assumptions about the probability of switching from one transparency regime to another). This implies that the truth-telling and signal-precision effects are even more important determinants of the effects of transparency on exchange rate misalignment.

If investors do not have common knowledge of the past, then the true steady-state equilibria are more difficult to characterize. In particular, the fact that investors learn ν_t forever once the foreign central bank makes an announcement implies that they will never again be perpetually disparately informed about interventions, even if higher-order expectations remain in equilibrium. This makes the equilibrium without transparency more similar to the equilibrium if investors have common knowledge of the past, although the importance of this past observation diminishes the longer the foreign central bank goes without making another announcement.

6.1 Common Knowledge of the Past

Suppose that in each period $t > 1$, the value of the previous period's intervention ν_{t-1} becomes common knowledge among all investors. In terms of the information set of investor i, this assumption implies that for all $t > 1$,

$$\begin{aligned} s_{it} &= s_{it-1} \cup \{x_{it}, y_{it}, e_t, \nu_{t-1}\}, \\ \tilde{s}_{it} &= \tilde{s}_{it-1} \cup \{x_{it}, y_{it}, e_t, \nu_t\}, \end{aligned} \tag{6.10}$$

and that $s_{i1} = \{x_{i1}, y_{i1}, e_1\}$ and $\tilde{s}_{i1} = \{x_{i1}, y_{i1}, e_1, \nu_1\}$. Note that this assumption ensures that the higher-order expectations from equation (6.2) collapse into more simple average expectations.

In the next section, I relax the assumption about public revelation of ν_{t-1} and also assume that the interest rate on peso bonds depends on a factor that is not perfectly observed. This creates an environment where higher-order expectations are an important part of the equilibrium steady state regardless of whether or not the foreign central bank announces the value of its intervention ν_t. In this case, the transitory demand of noise traders has persistent effects on the exchange rate. I demonstrate that transparency can magnify the persistent effect of this noise, in addition to magnifying its immediate effect as in this and the previous section's models.

This section's assumptions about the investors' information yield an equilibrium exchange rate that is similar to the two-period model analyzed in Section 2. In doing so, this section provides an interpretation of the exchange rate fundamentals from that benchmark model, with those fundamentals now equal to the time-discounted sum of spreads between foreign and domestic interest rates plus the time-discounted sum of risk premia. The discount factor is determined by the parameter $\alpha = \frac{1}{1+a}$, which measures the sensitivity of the foreign central bank's interest rate rule to deviations from the price target.

To better see this connection, recall that exchange rate fundamentals in the benchmark model are given by $f = \theta_f f_0 + \theta_\nu f_\nu$ (this is equation (2.2)), where f_0 represents the part of fundamentals that is unrelated to the foreign central bank's intervention and f_ν represents the part of fundamentals that is related to this intervention. The bank's interventions are independent of interest rates and other disturbances in this infinite-horizon setup, so $\theta_f f_0$ is replaced by the time-discounted sum of spreads between foreign and domestic interest rates (the first term in equation (6.2) from Lemma 6.2) and $\theta_\nu f_\nu$ is replaced by the time-discounted sum of risk premia (the second term in equation (6.2) from Lemma 6.2). As I show below, the extent of the relationship between the central bank's intervention and the time-discounted sum of risk premia in this setup is highly dependent on the persistence of interventions ρ_ν. Not surprisingly, then, this setup reproduces many of the two-period setup's predictions with ρ_ν replacing the parameter θ_ν.

I present the equilibrium exchange rate with no central bank announcement about ν_t before presenting the equilibrium exchange rate with a central bank announcement. These two cases are then compared, and the implications of transparency are stated and discussed. As always, I assume that an announcement by the foreign central bank is truthful and credible. All proofs from this section are in Section 7.

Proposition 6.3. *If in each period t the value of ν_{t-1} becomes common knowledge among all investors but the foreign central bank does not publicly announce the value of ν_t, then the steady-state equilibrium exchange rate is given by*

$$e_t = (\alpha - \rho_f \beta_f) f_t + (\psi_f + \beta_f) f_{t+1} - \rho_\nu \beta_\nu \nu_{t-1} + (\psi_\nu + \beta_\nu)\nu_t + \lambda \xi_t, \tag{6.11}$$

where $\psi_f = \frac{\alpha^2}{1-\alpha\rho_f}$, $\psi_\nu = \frac{\alpha\gamma\sigma^2}{1-\alpha\rho_\nu}$ *and* $\lambda, \beta_f, \beta_\nu$, *and* σ^2 *are such that*

$$\lambda = \frac{\lambda\psi_f(\psi_f+\beta_f)(\sigma_\eta^2+\sigma_\delta^2)\sigma_\epsilon^2\sigma_\zeta^2 + \lambda\alpha\rho_\nu\psi_\nu(\psi_\nu+\beta_\nu)(\sigma_\epsilon^2+\sigma_\zeta^2)\sigma_\eta^2\sigma_\delta^2}{\Psi} + \alpha\gamma\sigma^2, \tag{6.12}$$

$$\beta_f = \frac{\alpha\rho_\nu\psi_\nu(\psi_f+\beta_f)(\psi_\nu+\beta_\nu)\sigma_\epsilon^2\sigma_\eta^2\sigma_\delta^2 - \psi_f\left((\sigma_\eta^2+\sigma_\delta^2)\lambda^2\sigma_\xi^2 + (\psi_\nu+\beta_\nu)^2\sigma_\eta^2\sigma_\delta^2\right)\sigma_\epsilon^2}{\Psi}, \tag{6.13}$$

$$\beta_\nu = \frac{\psi_f(\psi_f+\beta_f)(\psi_\nu+\beta_\nu)\sigma_\epsilon^2\sigma_\eta^2\sigma_\delta^2 - \alpha\rho_\nu\psi_\nu\left((\sigma_\epsilon^2+\sigma_\zeta^2)\lambda^2\sigma_\xi^2 + (\psi_f+\beta_f)^2\sigma_\epsilon^2\sigma_\zeta^2\right)\sigma_\eta^2}{\Psi}, \tag{6.14}$$

$$\begin{aligned}\sigma^2 = {} & \frac{\psi_f^2}{\alpha^2}\sigma_\epsilon^2 + \rho_\nu^2\psi_\nu^2\sigma_\eta^2 + \lambda^2\sigma_\xi^2 + (\psi_f+\beta_f)^2\sigma_\zeta^2 + (\psi_\nu+\beta_\nu)^2\sigma_\delta^2 \\ & - \frac{\psi_f^2\sigma_\epsilon^4}{\alpha^2\Psi}\left[(\sigma_\eta^2+\sigma_\delta^2)\left(\lambda^2\sigma_\xi^2 + (\psi_f+\beta_f)^2\sigma_\zeta^2\right) + (\psi_\nu+\beta_\nu)^2\sigma_\eta^2\sigma_\delta^2\right] \\ & - \frac{\rho_\nu^2\psi_\nu^2\sigma_\eta^4}{\Psi}\left[(\sigma_\epsilon^2+\sigma_\zeta^2)\left(\lambda^2\sigma_\xi^2 + (\psi_\nu+\beta_\nu)^2\sigma_\delta^2\right) + (\psi_f+\beta_f)^2\sigma_\epsilon^2\sigma_\zeta^2\right] \\ & - \frac{2\rho_\nu\psi_f\psi_\nu}{\alpha\Psi}(\psi_f+\beta_f)(\psi_\nu+\beta_\nu)\sigma_\epsilon^2\sigma_\eta^2\sigma_\zeta^2\sigma_\delta^2,\end{aligned} \tag{6.15}$$

with $\Psi = (\psi_f+\beta_f)^2(\sigma_\eta^2+\sigma_\delta^2)\sigma_\epsilon^2\sigma_\zeta^2 + (\psi_\nu+\beta_\nu)^2(\sigma_\epsilon^2+\sigma_\zeta^2)\sigma_\eta^2\sigma_\delta^2 + (\sigma_\epsilon^2+\sigma_\zeta^2)(\sigma_\eta^2+\sigma_\delta^2)\lambda^2\sigma_\xi^2$.

If a real-valued solution to the system of equations given by Proposition 6.3 exists, then there exist two real solutions distinguished by the value of the steady-state variance σ^2. A thorough discussion of the viability of these multiple equilibria is beyond the scope of this paper, but in general, the high-variance equilibrium is not stable in the sense that any perceived deviation of the variance from this steady-state value generates an even larger actual deviation from that steady state.[29] With this instability in mind, I follow Bacchetta and van Wincoop (2006) and focus primarily on the low-variance steady-state equilibrium exchange rate. I emphasize that all of the results I present in Theorem 6.5 below apply also to the high-variance equilibria with and without transparency.

Equation (6.11) from Proposition 6.3 implies that the exchange rate in period $t+1$ is given by

$$\begin{aligned} e_{t+1} &= (\alpha - \rho_f\beta_f)f_{t+1} + (\psi_f+\beta_f)f_{t+2} - \rho_\nu\beta_\nu\nu_t + (\psi_\nu+\beta_\nu)\nu_{t+1} + \lambda\xi_{t+1} \\ &= \frac{\psi_f}{\alpha}f_{t+1} + \psi_\nu\rho_\nu\nu_t + \lambda\xi_{t+1} + (\psi_f+\beta_f)\zeta_{t+2} + (\psi_\nu+\beta_\nu)\delta_{t+1}. \end{aligned} \tag{6.16}$$

In the benchmark two-period model, the exchange rate in period two is given by $e_2 = f + \kappa$, with $f = \theta_f f_0 + \theta_\nu\nu$, so there are clearly similarities between that setup and this infinite-

[29]Consider any positive $\sigma_0^2 \neq \sigma^2$. One implication of this instability is that if investors observe past variances of the exchange rate and choose σ_t^2 in each period t as a weighted average of these past, observed variances, then σ_t^2 will never converge to the high-variance equilibrium value of σ^2.

horizon setup. In particular, equation (6.16) shows that this model's expression for e_{t+1} is the same as that model's expression for e_2, with θ_f replaced by $\frac{\psi_f}{\alpha}$, f_0 replaced by f_{t+1}, θ_ν replaced by $\rho_\nu\psi_\nu$, ν replaced by ν_t, and κ replaced by $\lambda\xi_{t+1} + (\psi_f + \beta_f)\zeta_{t+2} + (\psi_\nu + \beta_\nu)\delta_{t+1}$. As mentioned earlier, this model's transparency results are much like those from Section 2, with θ_ν now replaced by $\rho_\nu\psi_\nu$.

Before presenting these results, it is first necessary to characterize the steady-state equilibrium exchange rate when the foreign central bank makes a credible and truthful announcement of its intervention in period t. As in the benchmark model, let $\tilde{e}_t$ denote the exchange rate in period t if the central bank announces the value of ν_t to the investors in period t.

Proposition 6.4. *If in each period t the foreign central bank credibly and publicly announces the value of ν_t, then the steady-state equilibrium exchange rate is given by*

$$\tilde{e}_t = (\alpha - \rho_f\tilde{\beta}_f)f_t + (\psi_f + \tilde{\beta}_f)f_{t+1} + \psi_\nu\nu_t + \tilde{\lambda}\xi_t, \tag{6.17}$$

where $\psi_f = \frac{\alpha^2}{1-\alpha\rho_f}$, $\psi_\nu = \frac{\alpha\gamma\tilde{\sigma}^2}{1-\alpha\rho_\nu}$, and $\tilde{\lambda}, \tilde{\beta}_f$, and $\tilde{\sigma}^2$ are such that

$$\tilde{\lambda} = \frac{\tilde{\lambda}\psi_f(\psi_f + \tilde{\beta}_f)\sigma_\epsilon^2\sigma_\zeta^2}{(\psi_f + \tilde{\beta}_f)^2\sigma_\epsilon^2\sigma_\zeta^2 + (\sigma_\epsilon^2 + \sigma_\zeta^2)\tilde{\lambda}^2\sigma_\xi^2} + \alpha\gamma\tilde{\sigma}^2, \tag{6.18}$$

$$\tilde{\beta}_f = -\frac{\psi_f\sigma_\epsilon^2\tilde{\lambda}^2\sigma_\xi^2}{(\psi_f + \tilde{\beta}_f)^2\sigma_\epsilon^2\sigma_\zeta^2 + (\sigma_\epsilon^2 + \sigma_\zeta^2)\tilde{\lambda}^2\sigma_\xi^2}, \tag{6.19}$$

$$\tilde{\sigma}^2 = \frac{\psi_f^2}{\alpha^2}\sigma_\epsilon^2 + \tilde{\lambda}^2\sigma_\xi^2 + (\psi_f + \tilde{\beta}_f)^2\sigma_\zeta^2 + \psi_\nu^2\sigma_\delta^2 - \frac{\psi_f^2\sigma_\epsilon^4\left(\tilde{\lambda}^2\sigma_\xi^2 + (\psi_f + \tilde{\beta}_f)^2\sigma_\zeta^2\right)}{\alpha^2(\psi_f + \tilde{\beta}_f)^2\sigma_\epsilon^2\sigma_\zeta^2 + \alpha^2(\sigma_\epsilon^2 + \sigma_\zeta^2)\tilde{\lambda}^2\sigma_\xi^2}. \tag{6.20}$$

In this infinite-horizon model, investors know both the values of f_t and ν_{t-1} (and also ν_t in the case of transparency) and the stochastic processes for these variables. This implies that investors have common priors about the values of f_{t+1} and ν_t, a fact that shows up in Propositions 6.3 and 6.4 in the form of the parameters β_f, β_ν, and $\tilde{\beta}_f$. While these extra parameters complicate the equilibrium exchange rate expressions, the parameters λ and $\tilde{\lambda}$ still measure the extent of exchange rate misalignment as a result of noise traders' demand while the differences $\lambda - \alpha\gamma\sigma^2$ and $\tilde{\lambda} - \alpha\gamma\tilde{\sigma}^2$ still measure the bias of investors' expectations of fundamentals as a result of this demand.

Theorem 6.5. *The parameters* λ *and* $\tilde{\lambda}$ *satisfy*

$$\lim_{\sigma_\epsilon \to \infty} \lambda > \lim_{\sigma_\epsilon \to \infty} \tilde{\lambda} = 0, \qquad \lim_{\sigma_\xi \to 0} \lambda < \lim_{\sigma_\xi \to 0} \tilde{\lambda} = \infty,$$

$$\lim_{\sigma_\zeta \to 0} \lambda = \lim_{\sigma_\zeta \to 0} \tilde{\lambda} = 0, \qquad \lim_{\sigma_\delta \to 0} \lambda = \lim_{\sigma_\delta \to 0} \tilde{\lambda} > 0.$$

The limits of both λ and $\tilde{\lambda}$ as either σ_ξ, σ_ζ, or σ_δ increases to infinity are undefined since the systems of equations that define the steady-state equilibria cease to have real solutions in those limits. Theorem 6.5 establishes several comparative statics for the parameters λ and $\tilde{\lambda}$, many of which reproduce results from the benchmark model (see Theorem 2.4).

As shown by equation (6.16) above, the product $\rho_\nu \psi_\nu$ in this model replaces the parameter θ_ν from the two-period model of Section 2. This product is equal to the time-discounted sum of future risk premia, and the term ρ_ν measures the persistence of the foreign central bank's interventions and hence the extent to which an intervention in period t affects peso bond risk premia in future periods (more persistence implies more effect). Since future risk premia are part of exchange rate fundamentals, a higher value of ρ_ν implies that the central bank's intervention in period t has a larger effect on those fundamentals. In other words, the truth-telling effect of transparency is increasing in ρ_ν in this infinite-horizon model.

Figures 7 - 10 show that the parameter λ tends to be less than $\tilde{\lambda}$ for smaller values of ρ_ν and greater than $\tilde{\lambda}$ for larger values of ρ_ν. Note that Figure 7 is similar to the parameterization of the benchmark two-period model shown in Figure 3, and that all of these figures show that λ is increasing relative to $\tilde{\lambda}$ as the extent of information revealed by a central bank announcement increases. Although this section's parameterizations all generate standard deviations for changes in the exchange rate that are roughly consistent with what is observed in quarterly data, the spirit of these numerical exercises is to illustrate the mechanism by which exchange rate misalignment can be magnified rather than to create a quantitatively precise simulation. Indeed, all of the models that I discuss are highly stylized and intended to explore and characterize the interaction between the truth-telling and signal-precision effects of transparency rather than to produce a precise model of exchange rate determination.

The first, baseline parameterization, depicted in Figure 7, features a choice of parameters that yields an unconditional standard deviation of ten percent for changes in the exchange rate (this is roughly consistent with the data). The second parameterization, depicted by Figure 8, presents this same parameterization except the variance of investors' private signals about the central bank's intervention σ_η is smaller. This has the effect of bringing λ and $\tilde{\lambda}$ closer together. The third parameterization, depicted in Figure 9, presents the same parameterization as in Figure 8 except that now the unpredictability of noise traders σ_ξ is smaller. This has the effect of increasing both λ and $\tilde{\lambda}$. Finally, Figure 10 presents the same parameterization as in Figure 9 except that now the persistence of innovations in the interest rate on peso bonds is smaller (ρ_f is smaller). This has the effect of decreasing both λ and $\tilde{\lambda}$.

The behavior of λ and $\tilde{\lambda}$ in these figures is very similar to the behavior shown graphically in the benchmark model. Indeed, the main conclusion to draw from this infinite-horizon model with common knowledge of the past is that the results largely reproduce the results from the two-period model. This is important because it shows that the previous discussion about truth-telling and signal-precision effects of transparency and its implications for central bank intervention policy are perfectly consistent with a richer infinite-horizon setup.

6.2 Imperfect Common Knowledge of the Past

Suppose that the value of ν_{t-1} does not become common knowledge among all investors in period t. Suppose also that the interest rate on peso bonds in period t is now given by $i_t^* = ap_t^* + f_t + \chi_t + r$, where χ_t is i.i.d. normal with mean zero and variance σ_χ^2. Since investors only observe i_t^* and p_t^* in each period t, these assumptions imply that investors have imperfect common knowledge about the value of f_t and, if the central bank does not announce the size of its intervention, also about the value of ν_t. It follows that higher-order expectations are always part of the equilibrium exchange rate.

There have been a number of dynamic macroeconomic models that feature higher-order expectations, including the early models of Townsend (1983) and Singleton (1987), and more recently, the models of Bacchetta and van Wincoop (2006) and Lorenzoni (2009). With the exception of Townsend (1983), all of these setups cannot be solved directly and must instead be approximated. This is usually accomplished by assuming that the past exogenously becomes common knowledge with some lag, a technique that keeps the state space in these models finite and makes it possible to solve for the steady-state equilibrium using standard methods. There is, however, another technique for solving these models as described by Nimark (2011). Rather than assuming that the past becomes common knowledge, Nimark (2011) shows that the steady-state equilibrium of a model in which agents are perpetually disparately informed can be approximated arbitrarily well by exogenously bounding the order of agents' expectations. As this bound grows to infinity, the approximate equilibrium converges to the true equilibrium.

In this section, I use this technique to consider the equilibrium of this infinite-horizon model when investors do not have common knowledge of the past. In models with higher-order expectations such as this one, it is typical for transitory shocks to have permanent effects on the beliefs of agents, as shown by Allen et al. (2006), Bacchetta and van Wincoop (2008), Lorenzoni (2009), and Nimark (2012). Although these permanent effects diminish over time, they still introduce substantial excess volatility and disconnect between prices and fundamentals. The goal of this extension is to examine how the persistent effects of transitory changes in noise traders' demand for peso bonds compare with and without foreign central bank transparency. Consistent with all the other results in this paper, I find that central bank transparency often worsens the exchange rate misalignment caused by transitory shocks to noise traders' demand in the past. In these cases, persistent deviations of the exchange rate from its fundamental value are magnified by transparency.

Before presenting this section's results, it is necessary to introduce some notation. Let $\bar{i}_t = i_t^* - ap_t^* - r$, and note that in each period t, investors observe the common public signal $\bar{i}_t = f_t + \chi_t$ but are unable to infer the value of f_t because of the unobserved disturbance χ_t. Furthermore, in order to maintain symmetry and simplify the solution, suppose now that each investor i observes the private signal $x_{it} = f_t + \epsilon_{it}$ rather than the private signal $x_{it} = f_{t+1} + \epsilon_{it}$ in each period t.[30]

In terms of the information set of investor i, these assumptions imply that for all t,

$$\begin{aligned} s_{it} &= s_{it-1} \cup \{x_{it}, y_{it}, e_t, \bar{i}_t\}, \\ \tilde{s}_{it} &= \tilde{s}_{it-1} \cup \{x_{it}, y_{it}, e_t, \bar{i}_t, \nu_t\}, \end{aligned} \tag{6.21}$$

with $s_{i0} = \tilde{s}_{i0} = \varnothing$. Strictly speaking, the definition of a steady-state equilibrium exchange rate 6.1 must now be altered so that the aggregate state of the economy in each period t includes the disturbance χ_t and does not include f_{t+1} (it should instead include f_t). For the sake of brevity, I only mention these technical details rather than restating the full definition of equilibrium.

The equilibrium exchange rate in this setup is expressed as a function of higher-order expectations at time t only, so let $\overline{E}(0)_t[\cdot] = \cdot$, $\overline{E}(1)_t[\cdot] = \overline{E}_t[\cdot]$, and in general, $\overline{E}(j)_t[\cdot] = \overline{E}_t \overline{E}_t \cdots \overline{E}_t[\cdot]$ with the expectation repeated j times. For all $0 \le j \le k$, let

$$q_{jt} = \begin{pmatrix} \overline{E}(j)_t[f_t] & \overline{E}(j)_t[\nu_t] \end{pmatrix}', \tag{6.22}$$

and for all $t \in \mathrm{N}$, let

$$Q_t(k) = \begin{pmatrix} q_{0t}' & q_{1t}' & \cdots & q_{kt}' \end{pmatrix}', \tag{6.23}$$

$$w_t = \begin{pmatrix} \sigma_\zeta^{-1}\zeta_t & \sigma_\delta^{-1}\delta_t & \sigma_\chi^{-1}\chi_t & \sigma_\xi^{-1}\xi_t \end{pmatrix}'. \tag{6.24}$$

[30] This assumption is without loss of generality since $\rho_f x_{it} = \rho_f f_t + \rho_f \epsilon_{it} = f_{t+1} - \zeta_{t+1} + \rho_f \epsilon_{it}$, and hence a private signal of f_t is also a private signal of f_{t+1}.

Let $h_1 = \left(\begin{smallmatrix} 1 & 0 & 0 & \cdots \end{smallmatrix} \right)'$ and $h_2 = \left(\begin{smallmatrix} 0 & 1 & 0 & 0 & \cdots \end{smallmatrix} \right)'$, and let the matrix H be given by

$$H = \begin{pmatrix} \mathbf{0}_{2k+2\times 2} & \begin{matrix} I_{2k} \\ \mathbf{0}_{2\times 2k} \end{matrix} \end{pmatrix}, \tag{6.25}$$

where I_{2k} is equal to the identity matrix of dimension $2k$. This matrix evaluates the average expectation of a vector and then annihilates the highest-order expectation, so that

$$HQ_t(k) = \begin{pmatrix} q'_{1t} & q'_{2t} & \cdots & q'_{kt} & 0 & 0 \end{pmatrix}' = \begin{pmatrix} \overline{E}_t[q'_{0t}] & \overline{E}_t[q'_{1t}] & \cdots & \overline{E}_t[q'_{k-1t}] & 0 & 0 \end{pmatrix}'. \tag{6.26}$$

All proofs from this section are in Section 7.

Proposition 6.6. *Suppose that the interest rate on peso bonds is given by $i^*_t = ap^*_t + f_t + \chi_t + r$ in each period t. If in each period t the value of ν_{t-1} does not become common knowledge among all investors and the foreign central bank does not publicly announce the value of ν_t, then the steady-state equilibrium exchange rate is approximately given by the system of equations*

$$e_t = AQ_t(k) + \alpha\gamma\sigma^2\xi_t, \tag{6.27}$$

$$Q_t(k) = MQ_{t-1}(k) + Nw_t, \tag{6.28}$$

where the vector A satisfies

$$A = \sum_{n=0}^{\infty} \alpha^{n+1}(h'_1 + \gamma\sigma^2 h'_2)(MH)^n. \tag{6.29}$$

As the order of truncation k grows to infinity, the solution to this system of equations converges to the true steady-state equilibrium exchange rate.

If the foreign central bank announces the value of ν_t in period t, then investors continue to have imperfect common knowledge about f_t while commonly learning the value of ν_t. In order to characterize the equilibrium exchange rate in this case, it is necessary again to introduce more notation. For all $0 \leq j \leq k$, let $\tilde{q}_{jt} = \overline{E}(j)_t[f_t]$, and for all $t \in \mathrm{N}$, let

$$\tilde{Q}_t(k) = \begin{pmatrix} \tilde{q}_{0t} & \tilde{q}_{1t} & \cdots & \tilde{q}_{kt} \end{pmatrix}', \tag{6.30}$$

$$\tilde{H} = \begin{pmatrix} \mathbf{0}_{k+1\times 1} & \begin{matrix} I_k \\ \mathbf{0}_{1\times k} \end{matrix} \end{pmatrix}, \tag{6.31}$$

$$\tilde{w}_t = \begin{pmatrix} \sigma_\zeta^{-1}\zeta_t & \sigma_\chi^{-1}\chi_t & \sigma_\xi^{-1}\xi_t \end{pmatrix}'. \tag{6.32}$$

Proposition 6.7. *Suppose that the interest rate on peso bonds is given by $i_t^* = ap_t^* + f_t + \chi_t + r$ in each period t. If in each period t the foreign central bank credibly and publicly announces the value of ν_t, then the steady-state equilibrium exchange rate is approximately given by the system of equations*

$$\tilde{e}_t = \tilde{A}\tilde{Q}_t(k) + \frac{\alpha\gamma\tilde{\sigma}^2}{1-\alpha\rho_\nu}\nu_t + \alpha\gamma\tilde{\sigma}^2\xi_t, \tag{6.33}$$

$$\tilde{Q}_t(k) = \tilde{M}\tilde{Q}_{t-1}(k) + \tilde{N}\tilde{w}_t, \tag{6.34}$$

where the vector $\tilde{A}$ satisfies

$$\tilde{A} = \sum_{n=0}^{\infty} \alpha^{n+1} h_1'(\tilde{M}\tilde{H})^n. \tag{6.35}$$

As the order of truncation k grows to infinity, the solution to this system of equations converges to the true steady-state equilibrium exchange rate.

The matrices M and N and the steady-state variance σ^2 from Proposition 6.6 as well as the matrices $\tilde{M}$ and $\tilde{N}$ and the steady-state variance $\tilde{\sigma}^2$ from Proposition 6.7 must all be approximated numerically. They are determined by the solution to two systems of matrix equations as detailed in Section 7. As in Section 6.1, there are two solutions to both systems of equations, one corresponding to a high-variance steady state and the second corresponding to a low-variance steady state. Numerical approximations indicate that the high-variance steady state is unstable in the sense described earlier.

In Figure 11, I plot the response of the steady-state equilibrium exchange rates with and without transparency to a negative shock to the noise traders' demand for peso bonds in period t_0. This shock is normalized so that the exchange rate with transparency $\tilde{e}_t$ decreases five percent in period t_0. The persistent effect of this transitory shock is plotted over time. The parameterization shown in Figure 11 is similar to the baseline parameterization shown in Figure 7 from the previous section. The main difference is that the variance terms σ_ξ and σ_ζ in this section's figure are slightly smaller in order to compensate for the extra noise term χ_t and to keep the unconditional variance of changes in the exchange rate close to ten percent (which is roughly consistent with the data). In the parameterization shown in the figure, higher-order expectations are truncated at $k = 50$. I find that the results do not change if this is increased even further.

The message of Figure 11 is similar to the message of Section 6.1: transparency magnifies exchange rate misalignment for low values of ρ_ν, even if that misalignment arises from shocks to noise traders' demand for peso bonds in the past. In particular, this result is a generalization of the previous sections' result that $\tilde{\lambda} > \lambda$ since the equilibrium exchange rate in period t is now a function of $\xi_{t-1}, \xi_{t-2}, \ldots$ as well as ξ_t, and the multipliers on all of these noise terms are larger if the foreign central bank is transparent. More precisely, the exchange rate in period t is now of the form $e_t = \lambda\xi_t + \lambda_1\xi_{t-1} + \lambda_2\xi_{t-2} + \cdots$ (with a corresponding expression for $\tilde{e}_t$), and for low values of ρ_ν my numerical approximations demonstrate that $\tilde{\lambda} > \lambda$, $\tilde{\lambda}_1 > \lambda_1$, $\tilde{\lambda}_2 > \lambda_2$, and so on. One implication of this result is that periods of sustained exchange rate misalignment are likely to imply large differences between mispricing with and without transparency as the larger multipliers with either policy start to add up.

The policy implication of this setup with higher-order expectations is similar to the implication in all previous sections. If central bank announcements reveal sufficiently partial information about exchange rate fundamentals, then the truth-telling effect is likely to be smaller than the signal-precision effect and transparency is likely to exacerbate exchange rate misalignment. This section shows that this applies also to misalignment between the exchange rate and fundamentals in the future, since both the immediate and persistent effects of temporary disturbances are magnified in a similar manner.

7 Appendix: Proofs

This appendix presents the proofs of Propositions 5.1, 5.2, 6.3, 6.4, 6.6, and 6.7, and Theorems 5.3 and 6.5.

Proof of Proposition 5.1 Suppose that the exchange rate in period two is normally distributed conditional on investor i's information set. In a manner similar to the proofs of Propositions 2.2 and 2.3, it can be shown that market clearing in the peso bond market implies that $e_1 = \overline{E}_1[e_2] + \gamma\sigma^2(\nu + \xi)$. The equilibrium exchange rate is of the form $e_1 = f + \gamma\sigma^2\nu + \lambda\xi$, which by equation (5.1) implies that $e_1 = f + \gamma\sigma^2(a_\nu f_\nu + a_\xi\xi) + \lambda\xi$. It follows by standard Bayesian inference that the exchange rate in period two is normally distributed conditional on investor i's information set (this justifies the assumption of conditional normality) and that

$$\begin{pmatrix} E_{i1}[f_0] \\ E_{i1}[f_\nu] \end{pmatrix} = \begin{pmatrix} x_i \\ y_i \end{pmatrix} + \begin{pmatrix} \theta_f\sigma_\epsilon^2 \\ (\theta_\nu + a_\nu\gamma\sigma^2)\sigma_\eta^2 \end{pmatrix} \frac{e_1 - E_i[e_1]}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + a_\nu\gamma\sigma^2)^2\sigma_\eta^2 + (a_\xi\gamma\sigma^2 + \lambda)^2\sigma_\xi^2},$$

and hence also that

$$\begin{pmatrix} \overline{E}_1[f_0] \\ \overline{E}_1[f_\nu] \end{pmatrix} = \begin{pmatrix} f_0 \\ f_\nu \end{pmatrix} + \begin{pmatrix} \theta_f\sigma_\epsilon^2 \\ (\theta_\nu + a_\nu\gamma\sigma^2)\sigma_\eta^2 \end{pmatrix} \frac{(a_\xi\gamma\sigma^2 + \lambda)\xi}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + a_\nu\gamma\sigma^2)^2\sigma_\eta^2 + (a_\xi\gamma\sigma^2 + \lambda)^2\sigma_\xi^2}.$$

Substituting this last equation into the expression for the exchange rate in period one (recall that

$\overline{E}_1[e_2] = \overline{E}_1[f]$) yields

$$e_1 = f + \gamma\sigma^2\nu + \left(\frac{\theta_f^2(a_\xi\gamma\sigma^2 + \lambda)\sigma_\epsilon^2 + \theta_\nu(\theta_\nu + a_\nu\gamma\sigma^2)(a_\xi\gamma\sigma^2 + \lambda)\sigma_\eta^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + a_\nu\gamma\sigma^2)^2\sigma_\eta^2 + (a_\xi\gamma\sigma^2 + \lambda)^2\sigma_\xi^2} + \gamma\sigma^2\right)\xi. \tag{7.1}$$

The next step is to solve for σ^2, the conditional variance of the exchange rate in period two. Because $e_2 = \theta_f f_0 + \theta_\nu f_\nu + \kappa$, this conditional variance is given by $\sigma^2 = \theta_f^2\overline{\mathrm{Var}}_1[f_0] + \theta_\nu^2\overline{\mathrm{Var}}_1[f_\nu] + \sigma_\kappa^2 + 2\theta_f\theta_\nu\overline{\mathrm{Cov}}_1[f_0, f_\nu]$, just as in the earlier theorems' proofs. Bayesian inference implies that

$$\begin{aligned}\begin{pmatrix}\overline{\mathrm{Var}}_1[f_0] & \overline{\mathrm{Cov}}_1[f_0, f_\nu] \\ \overline{\mathrm{Cov}}_1[f_0, f_\nu] & \overline{\mathrm{Var}}_1[f_\nu]\end{pmatrix} &= \begin{pmatrix}\sigma_\epsilon^2 & 0 \\ 0 & \sigma_\eta^2\end{pmatrix} \\ &- \frac{1}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + a_\nu\gamma\sigma^2)^2\sigma_\eta^2 + (a_\xi\gamma\sigma^2 + \lambda)^2\sigma_\xi^2}\begin{pmatrix}\theta_f\sigma_\epsilon^2 \\ (\theta_\nu + a_\nu\gamma\sigma^2)\sigma_\eta^2\end{pmatrix}\begin{pmatrix}\theta_f\sigma_\epsilon^2 & (\theta_\nu + a_\nu\gamma\sigma^2)\sigma_\eta^2\end{pmatrix},\end{aligned}$$

so that

$$\overline{\mathrm{Var}}_1[f_0] = \sigma_\epsilon^2 - \frac{\theta_f^2\sigma_\epsilon^4}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + a_\nu\gamma\sigma^2)^2\sigma_\eta^2 + (a_\xi\gamma\sigma^2 + \lambda)^2\sigma_\xi^2},$$

$$\overline{\mathrm{Var}}_1[f_\nu] = \sigma_\eta^2 - \frac{(\theta_\nu + a_\nu\gamma\sigma^2)^2\sigma_\eta^4}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + a_\nu\gamma\sigma^2)^2\sigma_\eta^2 + (a_\xi\gamma\sigma^2 + \lambda)^2\sigma_\xi^2},$$

and

$$\overline{\mathrm{Cov}}_1[f_0, f_\nu] = \frac{-\theta_f(\theta_\nu + a_\nu\gamma\sigma^2)\sigma_\epsilon^2\sigma_\eta^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + a_\nu\gamma\sigma^2)^2\sigma_\eta^2 + (a_\xi\gamma\sigma^2 + \lambda)^2\sigma_\xi^2}.$$

It follows that

$$\sigma^2 = \theta_f^2\sigma_\epsilon^2 + \theta_\nu^2\sigma_\eta^2 + \sigma_\kappa^2 - \frac{\left(\theta_f^2\sigma_\epsilon^2 + \theta_\nu(\theta_\nu + a_\nu\gamma\sigma^2)\sigma_\eta^2\right)^2}{\theta_f^2\sigma_\epsilon^2 + (\theta_\nu + a_\nu\gamma\sigma^2)^2\sigma_\eta^2 + (a_\xi\gamma\sigma^2 + \lambda)^2\sigma_\xi^2}. \tag{7.2}$$

Note that this justifies the assumption that the conditional variance is equal for all investors i. The proof of existence is complete once we equate the undetermined coefficients λ and σ^2 with the implied expressions from equations (7.1) and (7.2). □

Proof of Proposition 5.2 Suppose that the exchange rate in period two is normally distributed conditional on investor i's information set. In a manner similar to the proofs of Propositions 2.2, 2.3, and 5.1, it can be shown that market clearing in the peso bond market implies that $\tilde{e}_1 = \overline{E}_1[e_2] + \gamma\tilde{\sigma}^2(\nu + \xi)$. The equilibrium exchange rate is of the form $\tilde{e}_1 = f + \gamma\tilde{\sigma}^2\nu + \tilde{\lambda}\xi$, which implies that $\tilde{e}_1 = f + \gamma\tilde{\sigma}^2(a_\nu f_\nu + a_\xi\xi) + \tilde{\lambda}\xi$. It follows by standard Bayesian inference that the exchange rate in period two is normally distributed conditional on investor i's information set (this justifies the assumption of conditional normality) and that

$$\begin{pmatrix}E_{i1}[f_0] \\ E_{i1}[f_\nu]\end{pmatrix} = \begin{pmatrix}x_i \\ y_i\end{pmatrix} + \begin{pmatrix}0 & \theta_f\sigma_\epsilon^2 \\ a_\nu\sigma_\eta^2 & \pi_\nu\sigma_\eta^2\end{pmatrix}\begin{pmatrix}a_\nu^2\sigma_\eta^2 + a_\xi^2\sigma_\xi^2 & a_\nu\pi_\nu\sigma_\eta^2 + a_\xi\pi_\xi\sigma_\xi^2 \\ a_\nu\pi_\nu\sigma_\eta^2 + a_\xi\pi_\xi\sigma_\xi^2 & \theta_f^2\sigma_\epsilon^2 + \pi_\nu^2\sigma_\eta^2 + \pi_\xi^2\sigma_\xi^2\end{pmatrix}^{-1}\begin{pmatrix}\nu - a_\nu y_i \\ \tilde{e}_1 - E_i[\tilde{e}_1]\end{pmatrix},$$

where $\pi_\nu = \theta_\nu + a_\nu\gamma\tilde{\sigma}^2$ and $\pi_\xi = a_\xi\gamma\tilde{\sigma}^2 + \tilde{\lambda}$. Let $\Psi = \theta_f^2(a_\nu^2\sigma_\eta^2 + a_\xi^2\sigma_\xi^2)\sigma_\epsilon^2 + (a_\nu\tilde{\lambda} - a_\xi\theta_\nu)^2\sigma_\eta^2\sigma_\xi^2$.

Averaging this last expression across all investors then yields

$$\begin{aligned}\begin{pmatrix}\overline{E}_1[f_0]\\ \overline{E}_1[f_\nu]\end{pmatrix} &= \begin{pmatrix}f_0\\ f_\nu\end{pmatrix} + \frac{1}{\Psi}\begin{pmatrix}0 & \theta_f\sigma_\epsilon^2\\ a_\nu\sigma_\eta^2 & \pi_\nu\sigma_\eta^2\end{pmatrix}\begin{pmatrix}\theta_f^2\sigma_\epsilon^2+\pi_\nu^2\sigma_\eta^2+\pi_\xi^2\sigma_\xi^2 & -a_\nu\pi_\nu\sigma_\eta^2-a_\xi\pi_\xi\sigma_\xi^2\\ -a_\nu\pi_\nu\sigma_\eta^2-a_\xi\pi_\xi\sigma_\xi^2 & a_\nu^2\sigma_\eta^2+a_\xi^2\sigma_\xi^2\end{pmatrix}\begin{pmatrix}a_\xi\xi\\ \pi_\xi\xi\end{pmatrix}\\ &= \begin{pmatrix}f_0\\ f_\nu\end{pmatrix} + \frac{1}{\Psi}\begin{pmatrix}-\theta_f\sigma_\epsilon^2(a_\nu\pi_\nu\sigma_\eta^2+a_\xi\pi_\xi\sigma_\xi^2) & \theta_f\sigma_\epsilon^2(a_\nu^2\sigma_\eta^2+a_\xi^2\sigma_\xi^2)\\ a_\nu\theta_f^2\sigma_\epsilon^2\sigma_\eta^2+\pi_\xi(a_\nu\pi_\xi-a_\xi\pi_\nu)\sigma_\eta^2\sigma_\xi^2 & a_\xi(a_\xi\pi_\nu-a_\nu\pi_\xi)\sigma_\eta^2\sigma_\xi^2\end{pmatrix}\begin{pmatrix}a_\xi\xi\\ \pi_\xi\xi\end{pmatrix}\\ &= \begin{pmatrix}f_0\\ f_\nu\end{pmatrix} + \frac{1}{\Psi}\begin{pmatrix}a_\nu\theta_f(a_\nu\tilde{\lambda}-a_\xi\theta_\nu)\sigma_\epsilon^2\sigma_\eta^2\xi\\ a_\nu a_\xi\theta_f^2\sigma_\epsilon^2\sigma_\eta^2\xi\end{pmatrix}.\end{aligned}$$

Finally, substituting the last equation into the expression for the exchange rate in period one implies that

$$\tilde{e}_1 = f + \gamma\tilde{\sigma}^2\nu + \left(\frac{\tilde{\lambda}a_\nu^2\theta_f^2\sigma_\epsilon^2\sigma_\eta^2}{\theta_f^2(a_\nu^2\sigma_\eta^2+a_\xi^2\sigma_\xi^2)\sigma_\epsilon^2+(a_\nu\tilde{\lambda}-a_\xi\theta_\nu)^2\sigma_\eta^2\sigma_\xi^2}+\gamma\tilde{\sigma}^2\right)\xi. \tag{7.3}$$

The next step is to solve for $\tilde{\sigma}^2$, the conditional variance of the exchange rate in period two. As in the proof of Proposition 5.1, this conditional variance is given by $\tilde{\sigma}^2 = \theta_f^2\overline{\mathrm{Var}}_1[f_0]+\theta_\nu^2\overline{\mathrm{Var}}_1[f_\nu]+\sigma_\kappa^2+2\theta_f\theta_\nu\overline{\mathrm{Cov}}_1[f_0,f_\nu]$. Bayesian inference implies that

$$\begin{aligned}\begin{pmatrix}\overline{\mathrm{Var}}_1[f_0] & \overline{\mathrm{Cov}}_1[f_0,f_\nu]\\ \overline{\mathrm{Cov}}_1[f_0,f_\nu] & \overline{\mathrm{Var}}_1[f_\nu]\end{pmatrix} &= \begin{pmatrix}\sigma_\epsilon^2 & 0\\ 0 & \sigma_\eta^2\end{pmatrix}\\ &\quad - \begin{pmatrix}0 & \theta_f\sigma_\epsilon^2\\ a_\nu\sigma_\eta^2 & \pi_\nu\sigma_\eta^2\end{pmatrix}\begin{pmatrix}a_\nu^2\sigma_\eta^2+a_\xi^2\sigma_\xi^2 & a_\nu\pi_\nu\sigma_\eta^2+a_\xi\pi_\xi\sigma_\xi^2\\ a_\nu\pi_\nu\sigma_\eta^2+a_\xi\pi_\xi\sigma_\xi^2 & \theta_f^2\sigma_\epsilon^2+\pi_\nu^2\sigma_\eta^2+\pi_\xi^2\sigma_\xi^2\end{pmatrix}^{-1}\begin{pmatrix}0 & a_\nu\sigma_\eta^2\\ \theta_f\sigma_\epsilon^2 & \pi_\nu\sigma_\eta^2\end{pmatrix},\end{aligned}$$

so that

$$\begin{aligned}\begin{pmatrix}\overline{\mathrm{Var}}_1[f_0] & \overline{\mathrm{Cov}}_1[f_0,f_\nu]\\ \overline{\mathrm{Cov}}_1[f_0,f_\nu] & \overline{\mathrm{Var}}_1[f_\nu]\end{pmatrix} &= \begin{pmatrix}\sigma_\epsilon^2 & 0\\ 0 & \sigma_\eta^2\end{pmatrix}\\ &\quad - \frac{1}{\Psi}\begin{pmatrix}-\theta_f\sigma_\epsilon^2(a_\nu\pi_\nu\sigma_\eta^2+a_\xi\pi_\xi\sigma_\xi^2) & \theta_f\sigma_\epsilon^2(a_\nu^2\sigma_\eta^2+a_\xi^2\sigma_\xi^2)\\ a_\nu\theta_f^2\sigma_\epsilon^2\sigma_\eta^2+\pi_\xi(a_\nu\pi_\xi-a_\xi\pi_\nu)\sigma_\eta^2\sigma_\xi^2 & a_\xi(a_\xi\pi_\nu-a_\nu\pi_\xi)\sigma_\eta^2\sigma_\xi^2\end{pmatrix}\begin{pmatrix}0 & a_\nu\sigma_\eta^2\\ \theta_f\sigma_\epsilon^2 & \pi_\nu\sigma_\eta^2\end{pmatrix}.\end{aligned}$$

It follows that

$$\begin{aligned}\overline{\mathrm{Var}}_1[f_0] &= \sigma_\epsilon^2 - \frac{\theta_f^2(a_\nu^2\sigma_\eta^2+a_\xi^2\sigma_\xi^2)\sigma_\epsilon^4}{\theta_f^2(a_\nu^2\sigma_\eta^2+a_\xi^2\sigma_\xi^2)\sigma_\epsilon^2+(a_\nu\tilde{\lambda}-a_\xi\theta_\nu)^2\sigma_\eta^2\sigma_\xi^2},\\ \overline{\mathrm{Var}}_1[f_\nu] &= \sigma_\eta^2 - \frac{a_\nu^2\theta_f^2\sigma_\epsilon^2\sigma_\eta^4+(a_\nu\tilde{\lambda}-a_\xi\theta_\nu)^2\sigma_\eta^4\sigma_\xi^2}{\theta_f^2(a_\nu^2\sigma_\eta^2+a_\xi^2\sigma_\xi^2)\sigma_\epsilon^2+(a_\nu\tilde{\lambda}-a_\xi\theta_\nu)^2\sigma_\eta^2\sigma_\xi^2},\end{aligned}$$

and

$$\overline{\mathrm{Cov}}_1[f_0,f_\nu] = \frac{a_\xi\theta_f(a_\nu\tilde{\lambda}-a_\xi\theta_\nu)\sigma_\epsilon^2\sigma_\eta^2\sigma_\xi^2}{\theta_f^2(a_\nu^2\sigma_\eta^2+a_\xi^2\sigma_\xi^2)\sigma_\epsilon^2+(a_\nu\tilde{\lambda}-a_\xi\theta_\nu)^2\sigma_\eta^2\sigma_\xi^2},$$

and hence also that

$$\tilde{\sigma}^2 = \theta_f^2\sigma_\epsilon^2+\theta_\nu^2\sigma_\eta^2+\sigma_\kappa^2 - \frac{a_\nu^2\theta_f^2(\theta_f^2\sigma_\epsilon^2+\theta_\nu^2\sigma_\eta^2)\sigma_\epsilon^2\sigma_\eta^2+\left(a_\xi\theta_f^2\sigma_\epsilon^2+\theta_\nu(a_\xi\theta_\nu-a_\nu\tilde{\lambda})\sigma_\eta^2\right)^2\sigma_\xi^2}{\theta_f^2(a_\nu^2\sigma_\eta^2+a_\xi^2\sigma_\xi^2)\sigma_\epsilon^2+(a_\nu\tilde{\lambda}-a_\xi\theta_\nu)^2\sigma_\eta^2\sigma_\xi^2}. \tag{7.4}$$

Note that this justifies the assumption that the conditional variance is equal for all investors i. The proof of existence is complete once we equate the undetermined coefficients $\tilde{\lambda}$ and $\tilde{\sigma}^2$ with the implied expressions from equations (7.3) and (7.4). $\square$

Proof of Theorem 5.3 Suppose that $\theta_\nu = 0$. According to equations (5.3) and (5.4), in this case λ and σ^2 are given by

$$\lambda = \frac{\theta_f^2(a_\xi\gamma\sigma^2 + \lambda)\sigma_\epsilon^2}{\theta_f^2\sigma_\epsilon^2 + (a_\nu\gamma\sigma^2)^2\sigma_\eta^2 + (a_\xi\gamma\sigma^2+\lambda)^2\sigma_\xi^2} + \gamma\sigma^2, \tag{7.5}$$

$$\sigma^2 = \sigma_\kappa^2 + \frac{\theta_f^2\sigma_\epsilon^2\left((a_\nu\gamma\sigma^2)^2\sigma_\eta^2 + (a_\xi\gamma\sigma^2+\lambda)^2\sigma_\xi^2\right)}{\theta_f^2\sigma_\epsilon^2 + (a_\nu\gamma\sigma^2)^2\sigma_\eta^2 + (a_\xi\gamma\sigma^2+\lambda)^2\sigma_\xi^2}, \tag{7.6}$$

and according to equations (5.6) and (5.7), in this case $\tilde{\lambda}$ and $\tilde{\sigma}_1^2$ are given by

$$\tilde{\lambda} = \frac{\tilde{\lambda}a_\nu^2\theta_f^2\sigma_\epsilon^2\sigma_\eta^2}{\theta_f^2(a_\nu^2\sigma_\eta^2 + a_\xi^2\sigma_\xi^2)\sigma_\epsilon^2 + a_\nu^2\tilde{\lambda}^2\sigma_\eta^2\sigma_\xi^2} + \gamma\tilde{\sigma}^2, \tag{7.7}$$

$$\tilde{\sigma}^2 = \sigma_\kappa^2 + \frac{a_\nu^2\theta_f^2\tilde{\lambda}^2\sigma_\epsilon^2\sigma_\eta^2\sigma_\xi^2}{\theta_f^2(a_\nu^2\sigma_\eta^2 + a_\xi^2\sigma_\xi^2)\sigma_\epsilon^2 + a_\nu^2\tilde{\lambda}^2\sigma_\eta^2\sigma_\xi^2}. \tag{7.8}$$

Consider now the limit as $a_\nu\sigma_\eta \to \infty$. As long as λ does not diverge to infinity, equations (7.5) and (7.6) imply that

$$\lim_{a_\nu\sigma_\eta\to\infty} \lambda = \lim_{a_\nu\sigma_\eta\to\infty} \gamma\sigma^2 = \gamma\sigma_\kappa^2 + \gamma\theta_f^2\sigma_\epsilon^2.$$

It is not difficult to show that the equilibrium equations imply that λ cannot diverge to infinity. In a similar manner, as long as $\tilde{\lambda}$ does not diverge to infinity, equations (7.7) and (7.8) imply that

$$\lim_{a_\nu\sigma_\eta\to\infty} \tilde{\lambda} = \lim_{a_\nu\sigma_\eta\to\infty} \frac{\tilde{\lambda}\theta_f^2\sigma_\epsilon^2}{\theta_f^2\sigma_\epsilon^2 + \tilde{\lambda}^2\sigma_\xi^2} + \gamma\tilde{\sigma}^2, \tag{7.9}$$

$$\lim_{a_\nu\sigma_\eta\to\infty} \tilde{\sigma}^2 = \lim_{a_\nu\sigma_\eta\to\infty} \sigma_\kappa^2 + \frac{\theta_f^2\tilde{\lambda}^2\sigma_\epsilon^2\sigma_\xi^2}{\theta_f^2\sigma_\epsilon^2 + \tilde{\lambda}^2\sigma_\xi^2}. \tag{7.10}$$

It is straightforward to show that the equilibrium conditions imply that $\tilde{\lambda}$ cannot diverge infinity, as well. Note that the equilibrium conditions given by equations (7.9) and (7.10) are identical to the equilibrium conditions given by equations (2.12) and (2.13) from Proposition 2.3 in Section 2, so that the parameters $\tilde{\lambda}$ and $\tilde{\sigma}_1^2$ in this model converge to the same value as the simpler model's parameters in the limit as $\theta_\nu \to 0$ and $a_\nu\sigma_\eta \to \infty$. Equations (7.9) and (7.10) together imply that

$$\lim_{a_\nu\sigma_\eta\to\infty} \tilde{\lambda}^3\sigma_\xi^2 = \lim_{a_\nu\sigma_\eta\to\infty} \gamma\sigma_\kappa^2(\theta_f^2\sigma_\epsilon^2 + \tilde{\lambda}^2\sigma_\xi^2) + \gamma\theta_f^2\tilde{\lambda}^2\sigma_\epsilon^2\sigma_\xi^2,$$

and hence that

$$\lim_{a_\nu\sigma_\eta\to\infty} \tilde{\lambda} = \lim_{a_\nu\sigma_\eta\to\infty} \gamma\sigma_\kappa^2\left(1 + \frac{\theta_f^2\sigma_\epsilon^2}{\tilde{\lambda}^2\sigma_\xi^2}\right) + \gamma\theta_f^2\sigma_\epsilon^2 > \gamma\sigma_\kappa^2 + \gamma\theta_f^2\sigma_\epsilon^2 = \lim_{a_\nu\sigma_\eta\to\infty} \lambda.$$

It follows by continuity, then, that there exist thresholds $\hat{\theta}_\nu > 0$ and $\widehat{a\sigma} > 0$ such that if $\theta_\nu < \hat{\theta}_\nu$ and $a_\nu \sigma_\eta > \widehat{a\sigma}$, then $\tilde{\lambda} > \lambda$.

Suppose that $\sigma_\eta = 0$. According to equations (5.3) and (5.4), in this case λ and σ^2 are given by

$$\lambda = \frac{\theta_f^2 (a_\xi \gamma \sigma^2 + \lambda) \sigma_\epsilon^2}{\theta_f^2 \sigma_\epsilon^2 + (a_\xi \gamma \sigma^2 + \lambda)^2 \sigma_\xi^2} + \gamma \sigma^2,$$
$$\sigma^2 = \sigma_\kappa^2 + \frac{\theta_f^2 (a_\xi \gamma \sigma^2 + \lambda)^2 \sigma_\epsilon^2 \sigma_\xi^2}{\theta_f^2 \sigma_\epsilon^2 + (a_\xi \gamma \sigma^2 + \lambda)^2 \sigma_\xi^2},$$

and according to equations (5.6) and (5.7), in this case $\tilde{\lambda} = \gamma \tilde{\sigma}^2 = \gamma \sigma_\kappa^2$. Because $\lambda > \gamma \sigma_\kappa^2 = \tilde{\lambda}$, it follows by continuity that there exists a threshold $\hat{\sigma}_\eta > 0$ such that if $\sigma_\eta < \hat{\sigma}_\eta$, then $\lambda > \tilde{\lambda}$.

In a similar manner, suppose that $a_\nu = 0$ (but now also $\sigma_\eta > 0$) and note that equations (5.3) and (5.4) imply that in this case λ and σ^2 are given by

$$\lambda = \frac{\theta_f^2 (a_\xi \gamma \sigma^2 + \lambda) \sigma_\epsilon^2 + \theta_\nu^2 (a_\xi \gamma \sigma^2 + \lambda) \sigma_\eta^2}{\theta_f^2 \sigma_\epsilon^2 + \theta_\nu^2 \sigma_\eta^2 + (a_\xi \gamma \sigma^2 + \lambda)^2 \sigma_\xi^2} + \gamma \sigma^2,$$
$$\sigma^2 = \sigma_\kappa^2 + \frac{(\theta_f^2 \sigma_\epsilon^2 + \theta_\nu^2 \sigma_\eta^2)(a_\xi \gamma \sigma^2 + \lambda)^2 \sigma_\xi^2}{\theta_f^2 \sigma_\epsilon^2 + \theta_\nu^2 \sigma_\eta^2 + (a_\xi \gamma \sigma^2 + \lambda)^2 \sigma_\xi^2}.$$

As in the previous case, equations (5.6) and (5.7) also imply that in this case $\tilde{\lambda} = \gamma \tilde{\sigma}^2 = \gamma \sigma_\kappa^2$. Because $\lambda > \gamma \sigma_\kappa^2 = \tilde{\lambda}$, it follows by continuity that there exists a threshold $\hat{a}_\nu$ such that if $a_\nu < \hat{a}_\nu$, then $\lambda > \tilde{\lambda}$. □

Proof of Proposition 6.3 Suppose that the steady-state equilibrium exchange rate in period $t+1$ is normally distributed conditional on investor i's information set in period t and that the conditional variance $\mathrm{Var}_{it}[e_{t+1}]$ is equal for all investors i (it must be equal in all periods t by definition). Lemma 6.2 then implies that the equilibrium exchange rate in period t must satisfy

$$e_t = \alpha f_t + \sum_{n=1}^{\infty} \alpha^{n+1} \overline{E}_t^n [f_{t+n}] + \gamma \sigma^2 \sum_{n=0}^{\infty} \alpha^{n+1} \overline{E}_t^n [\nu_{t+n}] + \alpha \gamma \sigma^2 \xi_t. \tag{7.11}$$

The exchange rate in period t is of the form

$$e_t = \alpha f_t + \psi_f f_{t+1} + \psi_\nu \nu_t + \lambda \xi_t + \beta_f \zeta_{t+1} + \beta_\nu \delta_t, \tag{7.12}$$

so the goal is to solve for the coefficients $\psi_f, \psi_\nu, \lambda, \beta_f$, and β_ν, which requires solving for the steady-state variance σ^2 as well.

The next step, then, is to solve for the average expectations $\overline{E}_t^n [f_{t+n}]$ and $\overline{E}_t^n [\nu_{t+n}]$. This requires first solving for the individual expectations $E_{it}[f_{t+1}]$ and $E_{it}[\nu_{t+1}]$, with the latter equal to $\rho_\nu E_{it}[\nu_t]$ since investors in period t have private signals of ν_t only. These expectations are more difficult to compute now that investors have prior distributions.

Let $E_{it}^0[\cdot]$, $\mathrm{Var}_{it}^0[\cdot]$, and $\mathrm{Cov}_{it}^0[\cdot]$ denote, respectively, the expected value, variance, and covariance with respect to the information set consisting only of f_t and the private signals x_{it} and y_{it}. If the form of the exchange rate in equation (7.12) is taken as given, then Bayesian inference implies both that the exchange rate in period $t+1$ is conditionally normally distributed (this justifies the assumption of conditional normality) and that

$$\begin{pmatrix} E_{it}[f_{t+1}] \\ E_{it}[\nu_t] \end{pmatrix} = \begin{pmatrix} x_{it} \\ y_{it} \end{pmatrix} + \begin{pmatrix} \sigma_\epsilon^2 & 0 & \psi_f \sigma_\epsilon^2 \\ 0 & \sigma_\eta^2 & \psi_\nu \sigma_\eta^2 \end{pmatrix} \begin{pmatrix} \sigma_\epsilon^2 + \sigma_\zeta^2 & 0 & \pi_f \\ 0 & \sigma_\eta^2 + \sigma_\delta^2 & \pi_\nu \\ \pi_f & \pi_\nu & \mathrm{Var}_{it}^0[e_t] \end{pmatrix}^{-1} \begin{pmatrix} \rho_f f_t - x_{it} \\ \rho_\nu \nu_{t-1} - y_{it} \\ e_t - E_{it}^0[e_t] \end{pmatrix},$$

where $\pi_f = \psi_f \sigma_\epsilon^2 - \beta_f \sigma_\zeta^2$ and $\pi_\nu = \psi_\nu \sigma_\eta^2 - \beta_\nu \sigma_\delta^2$. The inverse of the variance matrix in the above expression is equal to

$$\frac{1}{\Psi} \begin{pmatrix} (\sigma_\eta^2 + \sigma_\delta^2)\,\mathrm{Var}_{it}^0[e_t] - \pi_\nu^2 & \pi_f \pi_\nu & -(\sigma_\eta^2 + \sigma_\delta^2)\pi_f \\ \pi_f \pi_\nu & (\sigma_\epsilon^2 + \sigma_\zeta^2)\,\mathrm{Var}_{it}^0[e_t] - \pi_f^2 & -(\sigma_\epsilon^2 + \sigma_\zeta^2)\pi_\nu \\ -(\sigma_\eta^2 + \sigma_\delta^2)\pi_f & -(\sigma_\epsilon^2 + \sigma_\zeta^2)\pi_\nu & (\sigma_\epsilon^2 + \sigma_\zeta^2)(\sigma_\eta^2 + \sigma_\delta^2) \end{pmatrix}, \tag{7.13}$$

where

$$\begin{aligned} \Psi &= (\sigma_\epsilon^2 + \sigma_\zeta^2)(\sigma_\eta^2 + \sigma_\delta^2)\,\mathrm{Var}_{it}^0[e_t] - (\sigma_\eta^2 + \sigma_\delta^2)\pi_f^2 - (\sigma_\epsilon^2 + \sigma_\zeta^2)\pi_\nu^2 \\ &= (\sigma_\epsilon^2 + \sigma_\zeta^2)(\sigma_\eta^2 + \sigma_\delta^2)\lambda^2 \sigma_\xi^2 + (\psi_f^2 + 2\psi_f \beta_f + \beta_f^2)(\sigma_\eta^2 + \sigma_\delta^2)\sigma_\epsilon^2 \sigma_\zeta^2 + (\psi_\nu^2 + 2\psi_\nu \beta_\nu + \beta_\nu^2)(\sigma_\epsilon^2 + \sigma_\zeta^2)\sigma_\eta^2 \sigma_\delta^2 \\ &= (\psi_f + \beta_f)^2 (\sigma_\eta^2 + \sigma_\delta^2)\sigma_\epsilon^2 \sigma_\zeta^2 + (\psi_\nu + \beta_\nu)^2 (\sigma_\epsilon^2 + \sigma_\zeta^2)\sigma_\eta^2 \sigma_\delta^2 + (\sigma_\epsilon^2 + \sigma_\zeta^2)(\sigma_\eta^2 + \sigma_\delta^2)\lambda^2 \sigma_\xi^2. \end{aligned} \tag{7.14}$$

Note that $\overline{E}_t[x_{it}] = f_{t+1}$, $\overline{E}_t[y_{it}] = \nu_t$, and $\overline{E}_t[e_t - E_{it}^0[e_t]] = \lambda \xi_t + \beta_f \zeta_{t+1} + \beta_\nu \delta_t$, since $E\left[x_{it} \mid \mathcal{F}_t\right] = f_{t+1}$ and $E\left[y_{it} \mid \mathcal{F}_t\right] = \nu_t$ for all $i \in [0,1]$ and all $t \in \mathbb{N}$. Let

$$\Delta_f = \psi_f(\sigma_\epsilon^2 + \sigma_\zeta^2)(\sigma_\eta^2 + \sigma_\delta^2) - (\sigma_\eta^2 + \sigma_\delta^2)\pi_f = (\psi_f + \beta_f)(\sigma_\eta^2 + \sigma_\delta^2)\sigma_\zeta^2,$$

and

$$\Delta_\nu = \psi_\nu(\sigma_\epsilon^2 + \sigma_\zeta^2)(\sigma_\eta^2 + \sigma_\delta^2) - (\sigma_\epsilon^2 + \sigma_\zeta^2)\pi_\nu = (\psi_\nu + \beta_\nu)(\sigma_\epsilon^2 + \sigma_\zeta^2)\sigma_\delta^2.$$

Because $\mathrm{Var}_{it}^0[e_t] = \psi_f^2 \sigma_\epsilon^2 + \psi_\nu^2 \sigma_\eta^2 + \lambda^2 \sigma_\xi^2 + \beta_f^2 \sigma_\zeta^2 + \beta_\nu^2 \sigma_\delta^2$, it follows that

$$\begin{aligned} \overline{E}_t[f_{t+1}] &= f_{t+1} + \lambda \Delta_f \sigma_\epsilon^2 \xi_t + \frac{\sigma_\epsilon^2}{\Psi}\left((\sigma_\epsilon^2 + \sigma_\zeta^2)\pi_\nu \psi_f - \pi_f \pi_\nu + \beta_\nu \Delta_f\right)\delta_t \\ &\quad + \frac{\sigma_\epsilon^2}{\Psi}\left(\pi_\nu^2 + (\sigma_\eta^2 + \sigma_\delta^2)\pi_f \psi_f - (\sigma_\eta^2 + \sigma_\delta^2)\,\mathrm{Var}_{it}^0[e_t] + \beta_f \Delta_f\right)\zeta_{t+1} \\ &= f_{t+1} + \lambda \Delta_f \sigma_\epsilon^2 \xi_t + \frac{\sigma_\epsilon^2}{\Psi}\left((\psi_f + \beta_f)\sigma_\zeta^2 \pi_\nu + \beta_\nu \Delta_f\right)\delta_t \\ &\quad - \frac{\sigma_\epsilon^2}{\Psi}\left((\sigma_\eta^2 + \sigma_\delta^2)(\lambda^2 \sigma_\xi^2 + \beta_f^2 \sigma_\zeta^2 + \psi_f \beta_f \sigma_\zeta^2) + (\psi_\nu + \beta_\nu)^2 \sigma_\eta^2 \sigma_\delta^2 - \beta_f \Delta_f\right)\zeta_{t+1}, \end{aligned}$$

so that

$$\begin{aligned} \overline{E}_t[f_{t+1}] &= f_{t+1} + \lambda(\psi_f + \beta_f)(\sigma_\eta^2 + \sigma_\delta^2)\sigma_\epsilon^2 \sigma_\zeta^2 \xi_t \\ &\quad + \frac{(\psi_f + \beta_f)(\psi_\nu + \beta_\nu)\sigma_\epsilon^2 \sigma_\eta^2 \sigma_\zeta^2 \delta_t - \sigma_\epsilon^2\left((\sigma_\eta^2 + \sigma_\delta^2)\lambda^2 \sigma_\xi^2 + (\psi_\nu + \beta_\nu)^2 \sigma_\eta^2 \sigma_\delta^2\right)\zeta_{t+1}}{\Psi}. \end{aligned} \tag{7.15}$$

Similarly, it follows that

$$
\begin{aligned}
\overline{E}_t[\nu_t] &= \nu_t + \lambda\Delta_\nu\sigma_\eta^2\xi_t + \frac{\sigma_\epsilon^2}{\Psi}\left((\sigma_\eta^2+\sigma_\delta^2)\pi_f\psi_\nu - \pi_f\pi_\nu + \beta_f\Delta_\nu\right)\zeta_{t+1} \\
&\qquad + \frac{\sigma_\eta^2}{\Psi}\left(\pi_f^2 + (\sigma_\epsilon^2+\sigma_\zeta^2)\pi_\nu\psi_\nu - (\sigma_\epsilon^2+\sigma_\zeta^2)\operatorname{Var}_{it}^0[e_t] + \beta_\nu\Delta_\nu\right)\delta_t \\
&= \nu_t + \lambda\Delta_\nu\sigma_\eta^2\xi_t + \frac{\sigma_\epsilon^2}{\Psi}\left((\psi_\nu+\beta_\nu)\sigma_\delta^2\pi_f + \beta_f\Delta_\nu\right)\zeta_{t+1} \\
&\qquad - \frac{\sigma_\eta^2}{\Psi}\left((\sigma_\epsilon^2+\sigma_\zeta^2)(\lambda^2\sigma_\xi^2 + \beta_\nu^2\sigma_\delta^2 + \psi_\nu\beta_\nu\sigma_\delta^2) + (\psi_f+\beta_f)^2\sigma_\epsilon^2\sigma_\zeta^2 - \beta_\nu\Delta_\nu\right)\delta_t,
\end{aligned}
$$

so that

$$
\begin{aligned}
\overline{E}_t[\nu_t] &= \nu_t + \lambda(\psi_\nu+\beta_\nu)(\sigma_\epsilon^2+\sigma_\zeta^2)\sigma_\eta^2\sigma_\delta^2\xi_t \\
&\quad + \frac{(\psi_f+\beta_f)(\psi_\nu+\beta_\nu)\sigma_\epsilon^2\sigma_\eta^2\sigma_\delta^2\zeta_{t+1} - \sigma_\eta^2\left((\sigma_\epsilon^2+\sigma_\zeta^2)\lambda^2\sigma_\xi^2 + (\psi_f+\beta_f)^2\sigma_\epsilon^2\sigma_\zeta^2\right)\delta_t}{\Psi}.
\end{aligned}
\tag{7.16}
$$

Equations (7.15) and (7.16) state that both $\overline{E}_t[f_{t+1}]$ and $\overline{E}_t[\nu_t]$ are not functions of past noise trades or disturbances, so that higher-order beliefs collapse. More precisely, higher-order expectations are such that $\overline{E}_t^n[f_{t+n}] = \rho_f^{n-1}\overline{E}_t[f_{t+1}]$ and $\overline{E}_t^n[\nu_{t+n}] = \rho_\nu^n\overline{E}_t[\nu_t]$ for all $n > 1$. This important observation implies that the expression from equation (7.11) simplifies to

$$
\begin{aligned}
e_t &= \alpha f_t + \sum_{n=1}^{\infty}\alpha^{n+1}\rho_f^{n-1}\overline{E}_t[f_{t+1}] + \alpha\gamma\sigma^2\nu_t + \gamma\sigma^2\sum_{n=1}^{\infty}\alpha^{n+1}\rho_\nu^n\overline{E}_t[\nu_t] + \alpha\gamma\sigma^2\xi_t \\
&= \alpha f_t + \frac{\alpha^2}{1-\alpha\rho_f}\overline{E}_t[f_{t+1}] + \alpha\gamma\sigma^2\nu_t + \gamma\sigma^2\frac{\alpha^2\rho_\nu}{1-\alpha\rho_\nu}\overline{E}_t[\nu_t] + \alpha\gamma\sigma^2\xi_t.
\end{aligned}
\tag{7.17}
$$

Substituting equations (7.15) and (7.16) into equation (7.17) yields

$$
e_t = \alpha f_t + \psi_f f_{t+1} + \psi_\nu\nu_t + \lambda\xi_t + \beta_f\zeta_{t+1} + \beta_\nu\delta_t, \tag{7.18}
$$

where $\psi_f = \frac{\alpha^2}{1-\alpha\rho_f}$ and $\psi_\nu = \frac{\alpha\gamma\sigma^2}{1-\alpha\rho_\nu}$, and λ, β_f, and β_ν are given by the solution to equations (6.12), (6.13), and (6.14).

The final step is to solve for σ^2, the steady-state variance of the exchange rate, which is accomplished by first solving for $\overline{\operatorname{Var}}_t[f_{t+1}], \overline{\operatorname{Var}}_t[\nu_t]$, and $\overline{\operatorname{Cov}}_t[f_{t+1},\nu_t]$. Bayesian inference implies that

$$
\begin{aligned}
\begin{pmatrix} \overline{\operatorname{Var}}_t[f_{t+1}] & \overline{\operatorname{Cov}}_t[f_{t+1},\nu_t] \\ \overline{\operatorname{Cov}}_t[f_{t+1},\nu_t] & \overline{\operatorname{Var}}_t[\nu_t] \end{pmatrix} &= \begin{pmatrix} \sigma_\epsilon^2 & 0 \\ 0 & \sigma_\eta^2 \end{pmatrix} \\
&\quad - \begin{pmatrix} \sigma_\epsilon^2 & 0 & \psi_f\sigma_\epsilon^2 \\ 0 & \sigma_\eta^2 & \psi_\nu\sigma_\eta^2 \end{pmatrix} \begin{pmatrix} \sigma_\epsilon^2+\sigma_\zeta^2 & 0 & \pi_f \\ 0 & \sigma_\eta^2+\sigma_\delta^2 & \pi_\nu \\ \pi_f & \pi_\nu & \operatorname{Var}_{it}^0[e_t] \end{pmatrix}^{-1} \begin{pmatrix} \sigma_\epsilon^2 & 0 \\ 0 & \sigma_\eta^2 \\ \psi_f\sigma_\epsilon^2 & \psi_\nu\sigma_\eta^2 \end{pmatrix},
\end{aligned}
$$

where $\pi_f = \psi_f \sigma_\epsilon^2 - \beta_f \sigma_\zeta^2$ and $\pi_\nu = \psi_\nu \sigma_\eta^2 - \beta_\nu \sigma_\delta^2$ as before. It follows by equation (7.13) that

$$\begin{aligned}
\overline{\mathrm{Var}}_t[f_{t+1}] &= \sigma_\epsilon^2 - \frac{\sigma_\epsilon^4}{\Psi}\left((\sigma_\eta^2+\sigma_\delta^2)\,\mathrm{Var}^0_{it}[e_t] - \pi_\nu^2 - 2\psi_f(\sigma_\eta^2+\sigma_\delta^2)\pi_f + \psi_f^2(\sigma_\epsilon^2+\sigma_\zeta^2)(\sigma_\eta^2+\sigma_\delta^2)\right)\\
&= \sigma_\epsilon^2 - \frac{\sigma_\epsilon^4}{\Psi}\left[(\sigma_\eta^2+\sigma_\delta^2)\left(\psi_f^2\sigma_\epsilon^2+\psi_\nu^2\sigma_\eta^2+\lambda^2\sigma_\xi^2+\beta_f^2\sigma_\zeta^2+\beta_\nu^2\sigma_\delta^2-2\psi_f\pi_f+\psi_f^2(\sigma_\epsilon^2+\sigma_\zeta^2)\right)-\pi_\nu^2\right]\\
&= \sigma_\epsilon^2 - \frac{\sigma_\epsilon^4}{\Psi}\left[(\sigma_\eta^2+\sigma_\delta^2)\left(\psi_\nu^2\sigma_\eta^2+\lambda^2\sigma_\xi^2+(\psi_f+\beta_f)^2\sigma_\zeta^2+\beta_\nu^2\sigma_\delta^2\right)-(\psi_\nu\sigma_\eta^2-\beta_\nu\sigma_\delta^2)^2\right]\\
&= \sigma_\epsilon^2 - \frac{\sigma_\epsilon^4}{\Psi}\left[(\sigma_\eta^2+\sigma_\delta^2)\left(\lambda^2\sigma_\xi^2+(\psi_f+\beta_f)^2\sigma_\zeta^2\right)+(\psi_\nu+\beta_\nu)^2\sigma_\eta^2\sigma_\delta^2\right],
\end{aligned}$$

that

$$\begin{aligned}
\overline{\mathrm{Var}}_t[\nu_t] &= \sigma_\eta^2 - \frac{\sigma_\eta^4}{\Psi}\left((\sigma_\epsilon^2+\sigma_\zeta^2)\,\mathrm{Var}^0_{it}[e_t] - \pi_f^2 - 2\psi_\nu(\sigma_\epsilon^2+\sigma_\zeta^2)\pi_\nu + \psi_\nu^2(\sigma_\epsilon^2+\sigma_\zeta^2)(\sigma_\eta^2+\sigma_\delta^2)\right)\\
&= \sigma_\eta^2 - \frac{\sigma_\eta^4}{\Psi}\left[(\sigma_\epsilon^2+\sigma_\zeta^2)\left(\psi_f^2\sigma_\epsilon^2+\psi_\nu^2\sigma_\eta^2+\lambda^2\sigma_\xi^2+\beta_f^2\sigma_\zeta^2+\beta_\nu^2\sigma_\delta^2-2\psi_\nu\pi_\nu+\psi_\nu^2(\sigma_\eta^2+\sigma_\delta^2)\right)-\pi_f^2\right]\\
&= \sigma_\eta^2 - \frac{\sigma_\eta^4}{\Psi}\left[(\sigma_\epsilon^2+\sigma_\zeta^2)\left(\psi_f^2\sigma_\epsilon^2+\lambda^2\sigma_\xi^2+\beta_f^2\sigma_\zeta^2+(\psi_\nu+\beta_\nu)^2\sigma_\delta^2\right)-(\psi_f\sigma_\epsilon^2-\beta_f\sigma_\zeta^2)^2\right]\\
&= \sigma_\eta^2 - \frac{\sigma_\eta^4}{\Psi}\left[(\sigma_\epsilon^2+\sigma_\zeta^2)\left(\lambda^2\sigma_\xi^2+(\psi_\nu+\beta_\nu)^2\sigma_\delta^2\right)+(\psi_f+\beta_f)^2\sigma_\epsilon^2\sigma_\zeta^2\right],
\end{aligned}$$

and that

$$\begin{aligned}
\overline{\mathrm{Cov}}_t[f_{t+1},\nu_t] &= -\frac{\sigma_\epsilon^2\sigma_\eta^2}{\Psi}\left(\pi_f\pi_\nu - \psi_f(\sigma_\epsilon^2+\sigma_\zeta^2)\pi_\nu - \psi_\nu(\sigma_\eta^2+\sigma_\delta^2)\pi_f + \psi_f\psi_\nu(\sigma_\epsilon^2+\sigma_\zeta^2)(\sigma_\eta^2+\sigma_\delta^2)\right)\\
&= -\frac{\sigma_\epsilon^2\sigma_\eta^2}{\Psi}\left(\psi_\nu(\sigma_\eta^2+\sigma_\delta^2)(\psi_f+\beta_f)\sigma_\zeta^2 - \pi_\nu(\psi_f+\beta_f)\sigma_\zeta^2\right)\\
&= -\frac{(\psi_f+\beta_f)(\psi_\nu+\beta_\nu)\sigma_\epsilon^2\sigma_\eta^2\sigma_\zeta^2\sigma_\delta^2}{\Psi}.
\end{aligned}$$

As before, Ψ is given by equation (7.14). Equation (7.18) implies that the steady-state variance is equal to

$$\sigma^2 = \frac{\psi_f^2}{\alpha^2}\overline{\mathrm{Var}}_t[f_{t+1}] + \rho_\nu^2\psi_\nu^2\overline{\mathrm{Var}}_t[\nu_t] + \frac{2\rho_\nu\psi_f\psi_\nu}{\alpha}\overline{\mathrm{Cov}}_t[f_{t+1},\nu_t] + \lambda^2\sigma_\xi^2 + (\psi_f+\beta_f)^2\sigma_\zeta^2 + (\psi_\nu+\beta_\nu)^2\sigma_\delta^2,$$

which justifies the assumption that the conditional variance is equal for all investors i. Equation (6.15) follows. □

Proof of Proposition 6.4 This proof follows the proof of Proposition 6.3 very closely. Suppose that the steady-state equilibrium exchange rate in period $t+1$ is normally distributed conditional on investor i's information set in period t and that the conditional variance $\mathrm{Var}_{it}[\tilde{e}_{t+1}]$ is equal for all investors i. Lemma 6.2 then implies that the equilibrium exchange rate in period t satisfies equation (7.11). The exchange rate in period t is again of the form $\tilde{e}_t = \alpha f_t + \psi_f f_{t+1} + \psi_\nu \nu_t + \tilde{\lambda}\xi_t + \tilde{\beta}_f \zeta_{t+1}$ and the goal remains to solve for the coefficients $\psi_f, \psi_\nu, \tilde{\lambda}$, and $\tilde{\beta}_f$ as well as the conditional variance $\tilde{\sigma}^2$.

Bayesian inference again implies that the exchange rate in period $t+1$ is conditionally normally distributed, so the initial assumption is justified. As in the previous proof, $\overline{E}_t[x_{it}] = f_{t+1}$ and

$\overline{E}_t[e_t - E^0_{it}[e_t]] = \tilde{\lambda}\xi_t + \tilde{\beta}_f\zeta_{t+1}$. Furthermore, the average expectation of ν_t is equal to ν_t itself since the intervention is common knowledge, and so it follows that the average expectation of f_{t+1} is given by

$$\begin{aligned}\overline{E}_t[f_{t+1}] &= f_{t+1} + \begin{pmatrix}\sigma_\epsilon^2 & \psi_f\sigma_\epsilon^2\end{pmatrix}\begin{pmatrix}\sigma_\epsilon^2+\sigma_\zeta^2 & \psi_f\sigma_\epsilon^2-\tilde{\beta}_f\sigma_\zeta^2\\ \psi_f\sigma_\epsilon^2-\tilde{\beta}_f\sigma_\zeta^2 & \psi_f^2\sigma_\epsilon^2+\tilde{\lambda}^2\sigma_\xi^2+\tilde{\beta}_f^2\sigma_\zeta^2\end{pmatrix}^{-1}\begin{pmatrix}-\zeta_{t+1}\\ \tilde{\lambda}\xi_t+\tilde{\beta}_f\zeta_{t+1}\end{pmatrix}\\ &= f_{t+1} + \frac{1}{D}\begin{pmatrix}\sigma_\epsilon^2 & \psi_f\sigma_\epsilon^2\end{pmatrix}\begin{pmatrix}\psi_f^2\sigma_\epsilon^2+\tilde{\lambda}^2\sigma_\xi^2+\tilde{\beta}_f^2\sigma_\zeta^2 & \tilde{\beta}_f\sigma_\zeta^2-\psi_f\sigma_\epsilon^2\\ \tilde{\beta}_f\sigma_\zeta^2-\psi_f\sigma_\epsilon^2 & \sigma_\epsilon^2+\sigma_\zeta^2\end{pmatrix}\begin{pmatrix}-\zeta_{t+1}\\ \tilde{\lambda}\xi_t+\tilde{\beta}_f\zeta_{t+1}\end{pmatrix},\end{aligned}$$

where $D = (\psi_f+\tilde{\beta}_f)^2\sigma_\epsilon^2\sigma_\zeta^2 + (\sigma_\epsilon^2+\sigma_\zeta^2)\tilde{\lambda}^2\sigma_\xi^2$. It follows that

$$\begin{aligned}\overline{E}_t[f_{t+1}] &= f_{t+1} + \frac{1}{D}\begin{pmatrix}\left(\tilde{\lambda}^2\sigma_\xi^2+\tilde{\beta}_f(\psi_f+\tilde{\beta}_f)\sigma_\zeta^2\right)\sigma_\epsilon^2 & (\tilde{\beta}_f+\psi_f)\sigma_\epsilon^2\sigma_\zeta^2\end{pmatrix}\begin{pmatrix}-\zeta_{t+1}\\ \tilde{\lambda}\xi_t+\tilde{\beta}_f\zeta_{t+1}\end{pmatrix}\\ &= f_{t+1} + \frac{\tilde{\lambda}(\tilde{\beta}_f+\psi_f)\sigma_\epsilon^2\sigma_\zeta^2\xi_t - \tilde{\lambda}^2\sigma_\xi^2\sigma_\epsilon^2\zeta_{t+1}}{(\psi_f+\tilde{\beta}_f)^2\sigma_\epsilon^2\sigma_\zeta^2+(\sigma_\epsilon^2+\sigma_\zeta^2)\tilde{\lambda}^2\sigma_\xi^2}.\end{aligned} \tag{7.19}$$

Equation (7.19) states that $\overline{E}_t[f_{t+1}]$ is not a function of past noise trades or disturbances, so it follows that higher-order beliefs again collapse in this case. Furthermore, investors have no information about future values of ν_t besides knowledge of the current value of ν_t and the stochastic process that governs its motion. This implies that $\overline{E}^n_t[f_{t+n}] = \rho_f^{n-1}\overline{E}_t[f_{t+1}]$ and $\overline{E}^n_t[\nu_{t+n}] = \rho_\nu^n\nu_t$ for all $n > 1$, so that equation (7.11) simplifies to

$$\tilde{e}_t = \alpha f_t + \frac{\alpha^2}{1-\alpha\rho_f}\overline{E}_t[f_{t+1}] + \frac{\alpha\gamma\tilde{\sigma}^2}{1-\alpha\rho_\nu}\nu_t + \alpha\gamma\tilde{\sigma}^2\xi_t \tag{7.20}$$

Substituting equation (7.19) into equation (7.20) yields

$$\tilde{e}_t = \alpha f_t + \psi_f f_{t+1} + \psi_\nu\nu_t + \tilde{\lambda}\xi_t + \tilde{\beta}_f\zeta_{t+1}, \tag{7.21}$$

where $\psi_f = \frac{\alpha^2}{1-\alpha\rho_f}$ and $\psi_\nu = \frac{\alpha\gamma\tilde{\sigma}^2}{1-\alpha\rho_\nu}$, and $\tilde{\lambda}$ and $\tilde{\beta}_f$ are given by the solution to equations (6.18) and (6.19).

The final step of the proof is to solve for the steady-state variance of the exchange rate, $\tilde{\sigma}^2$. If investors know the value of ν_t in period t, then standard Bayesian inference implies that

$$\begin{aligned}\overline{\mathrm{Var}}_t[f_{t+1}] &= \sigma_\epsilon^2 - \begin{pmatrix}\sigma_\epsilon^2 & \psi_f\sigma_\epsilon^2\end{pmatrix}\begin{pmatrix}\sigma_\epsilon^2+\sigma_\zeta^2 & \psi_f\sigma_\epsilon^2-\tilde{\beta}_f\sigma_\zeta^2\\ \psi_f\sigma_\epsilon^2-\tilde{\beta}_f\sigma_\zeta^2 & \psi_f^2\sigma_\epsilon^2+\tilde{\lambda}^2\sigma_\xi^2+\tilde{\beta}_f^2\sigma_\zeta^2\end{pmatrix}^{-1}\begin{pmatrix}\sigma_\epsilon^2\\ \psi_f\sigma_\epsilon^2\end{pmatrix}\\ &= \sigma_\epsilon^2 - \frac{1}{D}\begin{pmatrix}\sigma_\epsilon^2 & \psi_f\sigma_\epsilon^2\end{pmatrix}\begin{pmatrix}\psi_f^2\sigma_\epsilon^2+\tilde{\lambda}^2\sigma_\xi^2+\tilde{\beta}_f^2\sigma_\zeta^2 & \tilde{\beta}_f\sigma_\zeta^2-\psi_f\sigma_\epsilon^2\\ \tilde{\beta}_f\sigma_\zeta^2-\psi_f\sigma_\epsilon^2 & \sigma_\epsilon^2+\sigma_\zeta^2\end{pmatrix}\begin{pmatrix}\sigma_\epsilon^2\\ \psi_f\sigma_\epsilon^2\end{pmatrix}\\ &= \sigma_\epsilon^2 - \frac{\sigma_\epsilon^2}{D}\begin{pmatrix}\tilde{\lambda}^2\sigma_\xi^2+\tilde{\beta}_f(\psi_f+\tilde{\beta}_f)\sigma_\zeta^2 & (\psi_f+\tilde{\beta}_f)\sigma_\zeta^2\end{pmatrix}\begin{pmatrix}\sigma_\epsilon^2\\ \psi_f\sigma_\epsilon^2\end{pmatrix}\\ &= \sigma_\epsilon^2 - \frac{\sigma_\epsilon^4\left(\tilde{\lambda}^2\sigma_\xi^2+(\psi_f+\tilde{\beta}_f)^2\sigma_\zeta^2\right)}{(\psi_f+\tilde{\beta}_f)^2\sigma_\epsilon^2\sigma_\zeta^2+(\sigma_\epsilon^2+\sigma_\zeta^2)\tilde{\lambda}^2\sigma_\xi^2}.\end{aligned}$$

Equation (7.21) implies that the steady-state variance is equal to

$$\tilde{\sigma}^2 = \frac{\psi_f^2}{\alpha^2}\overline{\mathrm{Var}}_t[f_{t+1}] + \tilde{\lambda}^2\sigma_\xi^2 + (\psi_f + \tilde{\beta}_f)^2\sigma_\zeta^2 + \psi_\nu^2\sigma_\delta^2,$$

which justifies the assumption that the conditional variance is equal for all investors i. Equation (6.20) follows. □

Proof of Theorem 6.5 Let $\tilde{\Psi} = (\psi_f + \tilde{\beta}_f)^2\sigma_\epsilon^2\sigma_\zeta^2 + (\sigma_\epsilon^2 + \sigma_\zeta^2)\tilde{\lambda}^2\sigma_\xi^2$, and recall that

$$\frac{\Psi}{\sigma_\eta^2 + \sigma_\delta^2} = (\psi_f + \beta_f)^2\sigma_\epsilon^2\sigma_\zeta^2 + (\psi_\nu + \beta_\nu)^2(\sigma_\epsilon^2 + \sigma_\zeta^2)\frac{\sigma_\eta^2\sigma_\delta^2}{\sigma_\eta^2 + \sigma_\delta^2} + (\sigma_\epsilon^2 + \sigma_\zeta^2)\lambda^2\sigma_\xi^2. \tag{7.22}$$

According to equations (6.15) and (6.20),

$$\begin{aligned}\sigma^2 = {} & \frac{\psi_f^2\sigma_\epsilon^2\sigma_\zeta^2\left((\sigma_\eta^2 + \sigma_\delta^2)\lambda^2\sigma_\xi^2 + (\psi_\nu + \beta_\nu)^2\sigma_\eta^2\sigma_\delta^2\right)}{\alpha^2\Psi} + \frac{\rho_\nu^2\psi_\nu^2\sigma_\eta^2\sigma_\delta^2\left((\sigma_\epsilon^2 + \sigma_\zeta^2)\lambda^2\sigma_\xi^2 + (\psi_f + \beta_f)^2\sigma_\epsilon^2\sigma_\zeta^2\right)}{\Psi} \\ & + \lambda^2\sigma_\xi^2 + (\psi_f + \beta_f)^2\sigma_\zeta^2 + (\psi_\nu + \beta_\nu)^2\sigma_\delta^2 - \frac{2\rho_\nu\psi_f\psi_\nu}{\alpha\Psi}(\psi_f + \beta_f)(\psi_\nu + \beta_\nu)\sigma_\epsilon^2\sigma_\eta^2\sigma_\zeta^2\sigma_\delta^2\end{aligned} \tag{7.23}$$

and

$$\tilde{\sigma}^2 = \frac{\psi_f^2\sigma_\epsilon^2\sigma_\zeta^2\tilde{\lambda}^2\sigma_\xi^2}{\alpha^2\tilde{\Psi}} + \tilde{\lambda}^2\sigma_\xi^2 + (\psi_f + \tilde{\beta}_f)^2\sigma_\zeta^2 + \psi_\nu^2\sigma_\delta^2. \tag{7.24}$$

Throughout this proof, I assume that the parameters of the model are such that there exist real solutions λ and $\tilde{\lambda}$ to the systems of equations given by Propositions 6.3 and 6.4. If this is not the case, then these limits are undefined.

Consider the limit of $\lambda, \tilde{\lambda}$ as $\sigma_\xi \to 0$ and suppose that $\tilde{\lambda}$ does not diverge to infinity. In this case, $\tilde{\lambda}^2\sigma_\xi^2 \to 0$ so that by equations (6.18) and (6.19) it follows that $\tilde{\beta}_f \to 0$ and $\lim_{\sigma_\xi \to 0}\tilde{\lambda} = \lim_{\sigma_\xi \to 0}\tilde{\lambda} + \alpha\gamma\tilde{\sigma}^2$. Of course, the limit of $\tilde{\lambda}$ and $\tilde{\lambda} + \alpha\gamma\tilde{\sigma}^2$ can only be equal if either $\tilde{\lambda} \to 0$ or $\tilde{\lambda} \to \infty$. Equation (7.24) implies that $\tilde{\sigma}^2 \geq \psi_f^2\sigma_\zeta^2 > 0$ in the limit, so it must be that $\tilde{\lambda} \to \infty$ as $\sigma_\xi \to 0$. On the other hand, if λ does not diverge to infinity as $\sigma_\xi \to 0$, then equations (7.22), (6.12), and (7.23) imply that

$$\lim_{\sigma_\xi \to 0}\lambda = \lim_{\sigma_\xi \to 0}\frac{\lambda\psi_f(\psi_f + \beta_f)(\sigma_\eta^2 + \sigma_\delta^2)\sigma_\epsilon^2\sigma_\delta^2 + \lambda\alpha\rho_\nu\psi_\nu(\psi_\nu + \beta_\nu)(\sigma_\epsilon^2 + \sigma_\zeta^2)\sigma_\eta^2\sigma_\delta^2}{(\psi_f + \beta_f)^2(\sigma_\eta^2 + \sigma_\delta^2)\sigma_\epsilon^2\sigma_\zeta^2 + (\psi_\nu + \beta_\nu)^2(\sigma_\epsilon^2 + \sigma_\zeta^2)\sigma_\eta^2\sigma_\delta^2} + \alpha\gamma\sigma^2,$$

with

$$\begin{aligned}\lim_{\sigma_\xi \to 0} & \frac{\psi_f^2\sigma_\epsilon^2\sigma_\zeta^2(\psi_\nu + \beta_\nu)^2\sigma_\eta^2\sigma_\delta^2 + \alpha^2\rho_\nu^2\psi_\nu^2\sigma_\eta^2\sigma_\delta^2(\psi_f + \beta_f)^2\sigma_\epsilon^2\sigma_\zeta^2 - 2\alpha\rho_\nu\psi_f\psi_\nu(\psi_f + \beta_f)(\psi_\nu + \beta_\nu)\sigma_\epsilon^2\sigma_\eta^2\sigma_\zeta^2\sigma_\delta^2}{\alpha^2(\psi_f + \beta_f)^2(\sigma_\eta^2 + \sigma_\delta^2)\sigma_\epsilon^2\sigma_\zeta^2 + \alpha^2(\psi_\nu + \beta_\nu)^2(\sigma_\epsilon^2 + \sigma_\zeta^2)\sigma_\eta^2\sigma_\delta^2} \\ & + (\psi_f + \beta_f)^2\sigma_\zeta^2 + (\psi_\nu + \beta_\nu)^2\sigma_\delta^2 = \lim_{\sigma_\xi \to 0}\sigma^2.\end{aligned}$$

As long as $\sigma_\eta > 0$, it follows that λ converges to a finite limit.

Consider the limit of $\lambda, \tilde{\lambda}$ as $\sigma_\epsilon \to \infty$. If $\tilde{\lambda}$ converges to a finite limit in this case, then equation

(6.19) implies that $\tilde{\beta}_f \to -\psi_f$ so that $\lim_{\sigma_\epsilon \to \infty} \tilde{\lambda} = \lim_{\sigma_\epsilon \to \infty} \alpha\gamma\tilde{\sigma}^2$. Equation (7.24) implies that

$$\lim_{\sigma_\epsilon \to \infty} \tilde{\sigma}^2 = \lim_{\sigma_\epsilon \to \infty} \tilde{\lambda}^2 \sigma_\xi^2 + \psi_\nu^2 \sigma_\delta^2 = \lim_{\sigma_\epsilon \to \infty} \alpha^2\gamma^2\tilde{\sigma}^4\sigma_\xi^2 + \frac{\alpha^2\gamma^2\tilde{\sigma}^4}{(1-\alpha\rho_\nu)^2}\sigma_\delta^2.$$

The only real solution to the equation $\tilde{\sigma}^2 = \alpha^2\gamma^2\tilde{\sigma}^4\sigma_\xi^2 + \frac{\alpha^2\gamma^2\tilde{\sigma}^4}{(1-\alpha\rho_\nu)^2}\sigma_\delta^2$ is $\tilde{\sigma}^2 = 0$, so it follows that both $\tilde{\sigma}^2 \to 0$ and $\tilde{\lambda} \to 0$ as $\sigma_\epsilon \to \infty$. According to equation (7.22),

$$\lim_{\sigma_\epsilon \to \infty} \frac{\Psi}{\sigma_\epsilon^2} = \lim_{\sigma_\epsilon \to \infty} (\psi_f + \beta_f)^2(\sigma_\eta^2 + \sigma_\delta^2)\sigma_\zeta^2 + (\psi_\nu + \beta_\nu)^2\sigma_\eta^2\sigma_\delta^2 + (\sigma_\eta^2 + \sigma_\delta^2)\lambda^2\sigma_\xi^2,$$

so that, much like in the case of $\tilde{\beta}_f$, equation (6.13) implies that $\beta_f \to -\psi_f$ as $\sigma_\epsilon \to \infty$. These properties imply that

$$\lim_{\sigma_\epsilon \to \infty} \lambda = \lim_{\sigma_\epsilon \to \infty} \frac{\lambda\alpha\rho_\nu\psi_\nu(\psi_\nu + \beta_\nu)\sigma_\eta^2\sigma_\delta^2}{(\psi_\nu + \beta_\nu)^2\sigma_\eta^2\sigma_\delta^2 + (\sigma_\eta^2 + \sigma_\delta^2)\lambda^2\sigma_\xi^2} + \alpha\gamma\sigma^2. \tag{7.25}$$

The key equation is equation (6.14), which implies that

$$\lim_{\sigma_\epsilon \to \infty} \beta_\nu = \lim_{\sigma_\epsilon \to \infty} \frac{-\alpha\rho_\nu\psi_\nu\sigma_\eta^2\lambda^2\sigma_\xi^2}{(\psi_\nu + \beta_\nu)^2\sigma_\eta^2\sigma_\delta^2 + (\sigma_\eta^2 + \sigma_\delta^2)\lambda^2\sigma_\xi^2} + \alpha\gamma\sigma^2,$$

so that $\psi_\nu + \beta_\nu$ does not converge to zero since $\alpha\rho_\nu < 1$. All that remains is to show that σ^2 and hence ψ_ν does not converge to zero as $\sigma_\epsilon \to \infty$. This follows by equation (7.23), which implies that

$$\lim_{\sigma_\epsilon \to \infty} \sigma^2 = \lim_{\sigma_\epsilon \to \infty} \frac{\psi_f^2}{\alpha^2}\sigma_\zeta^2 + \rho_\nu^2\psi_\nu^2\sigma_\eta^2\sigma_\delta^2 + \lambda^2\sigma_\xi^2 + (\psi_\nu + \beta_\nu)^2\sigma_\delta^2. \tag{7.26}$$

The solution to this equation in the limit must be greater than zero since it contains the constant term $\frac{\psi_f^2}{\alpha^2}\sigma_\zeta^2 > 0$. It follows by equation (7.25) that λ converges to a constant greater than zero as $\sigma_\epsilon \to \infty$.

Consider the limit of $\lambda, \tilde{\lambda}$ as $\sigma_\zeta \to 0$. As in the case of $\sigma_\epsilon \to \infty$, equation (6.19) implies that $\tilde{\beta}_f \to -\psi_f$ in this case and hence by equation (7.24) it follows that $\tilde{\sigma}^2 \to 0$ and $\tilde{\lambda} \to 0$. It is not difficult to show that a limit equation identical to equation (7.25) obtains for this case where $\sigma_\zeta \to 0$, and that a similar equation to equation (7.26) also obtains. The key difference, however, is that if $\sigma_\zeta \to 0$, equation (7.26) changes so that

$$\lim_{\sigma_\zeta \to 0} \sigma^2 = \lim_{\sigma_\zeta \to 0} \rho_\nu^2\psi_\nu^2\sigma_\eta^2\sigma_\delta^2 + \lambda^2\sigma_\xi^2 + (\psi_\nu + \beta_\nu)^2\sigma_\delta^2,$$

and hence both σ^2 and ψ_ν converge to zero in the limit. It follows by equation (7.25) that $\lambda \to 0$ as $\sigma_\zeta \to 0$.

Consider the limit of $\lambda, \tilde{\lambda}$ as $\sigma_\delta \to 0$. Equation (7.22) implies that

$$\lim_{\sigma_\delta \to 0} \Psi = \lim_{\sigma_\delta \to 0} (\psi_f + \beta_f)^2\sigma_\epsilon^2\sigma_\zeta^2\sigma_\eta^2 + (\sigma_\epsilon^2 + \sigma_\zeta^2)\sigma_\eta^2\lambda^2\sigma_\xi^2,$$

and hence equations (6.12) and (7.23) imply that

$$\lim_{\sigma_\delta \to 0} \lambda = \lim_{\sigma_\delta \to 0} \frac{\lambda \psi_f(\psi_f + \beta_f)\sigma_\epsilon^2 \sigma_\zeta^2}{(\psi_f + \beta_f)^2 \sigma_\epsilon^2 \sigma_\zeta^2 + (\sigma_\epsilon^2 + \sigma_\zeta^2)\lambda^2 \sigma_\xi^2},$$

and

$$\lim_{\sigma_\delta \to 0} \sigma^2 = \lim_{\sigma_\delta \to 0} \frac{\psi_f^2 \sigma_\epsilon^2 \sigma_\zeta^2 \lambda^2 \sigma_\xi^2}{\alpha^2(\psi_f + \beta_f)^2 \sigma_\epsilon^2 \sigma_\zeta^2 + \alpha^2(\sigma_\epsilon^2 + \sigma_\zeta^2)\lambda^2 \sigma_\xi^2} + \lambda^2 \sigma_\xi^2 + (\psi_f + \beta_f)^2 \sigma_\zeta^2.$$

Equation (6.13) also implies that

$$\lim_{\sigma_\delta \to 0} \tilde{\beta}_f = \lim_{\sigma_\delta \to 0} -\frac{\psi_f \sigma_\epsilon^2 \lambda^2 \sigma_\xi^2}{(\psi_f + \beta_f)^2 \sigma_\epsilon^2 \sigma_\zeta^2 + (\sigma_\epsilon^2 + \sigma_\zeta^2)\lambda^2 \sigma_\xi^2}.$$

Meanwhile, equations (6.18), (6.19), and (7.24) imply that an identical set of equations jointly determine the value of $\tilde{\lambda}$ as $\sigma_\delta \to 0$, so it follows that $\lim_{\sigma_\delta \to 0} \lambda = \lim_{\sigma_\delta \to 0} \tilde{\lambda}$. □

Proof of Proposition 6.6 Suppose that the steady-state equilibrium exchange rate in period $t+1$ is normally distributed conditional on investor i's information set in period t. Suppose also that the conditional variance $\mathrm{Var}_{it}[e_{t+1}]$ is equal for all investors i. Lemma 6.2 then implies that the equilibrium exchange rate in period t must satisfy

$$e_t = \sum_{n=0}^{\infty} \alpha^{n+1} \overline{E}_t^n [f_{t+n}] + \gamma \sigma^2 \sum_{n=0}^{\infty} \alpha^{n+1} \overline{E}_t^n [\nu_{t+n}] + \alpha \gamma \sigma^2 \xi_t. \tag{7.27}$$

The exchange rate in period t is of the form

$$e_t = A Q_t(k) + \alpha \gamma \sigma^2 \xi_t, \tag{7.28}$$

$$Q_t(k) = M Q_{t-1}(k) + N w_t, \tag{7.29}$$

where $k > 0$ is the level at which higher-order expectations are truncated in the model. The goal is to solve for the equilibrium conditions that characterize the matrices M and N, the vector A, and the steady-state variance σ^2.

The definitions of the higher-order expectations vector $Q_t(k)$ and the matrices M and N imply that $\overline{E}_t^n[f_{t+n}] = h_1'(MH)^n Q_t(k)$ and $\overline{E}_t^n[\nu_{t+n}] = h_2'(MH)^n Q_t(k)$ for all $n \geq 1$. Equation (7.27) then implies that

$$e_t = \sum_{n=0}^{\infty} \alpha^{n+1}(h_1' + \gamma \sigma^2 h_2')(MH)^n Q_t(k) + \alpha \gamma \sigma^2 \xi_t,$$

so it follows by equation (7.28) that the vector A must satisfy

$$A = \sum_{n=0}^{\infty} \alpha^{n+1}(h_1' + \gamma \sigma^2 h_2')(MH)^n.$$

Note that this equation matches equation (6.29) exactly, so that all that remains of this proof is to characterize the state transition matrices M and N and the steady-state variance σ^2.

Recall that $\bar{i}_t = i_t^* - ap_t^* - r = f_t + \chi_t$. In each period t, each investor i observes

$$z_{it} = \begin{pmatrix} x_{it} \\ y_{it} \\ \bar{i}_t \\ e_t \end{pmatrix} = DQ_t(k) + R \begin{pmatrix} \sigma_\epsilon^{-1}\epsilon_{it} \\ \sigma_\eta^{-1}\eta_{it} \\ \sigma_\zeta^{-1}\zeta_t \\ \sigma_\delta^{-1}\delta_t \\ \sigma_\chi^{-1}\chi_t \\ \sigma_\xi^{-1}\xi_t \end{pmatrix},$$

where

$$D = \begin{pmatrix} 1 & 0 & \\ 0 & 1 & \mathbf{0}_{3\times 2k} \\ 1 & 0 & \\ & & A \end{pmatrix},$$

and $R = \begin{pmatrix} R_1 & R_2 \end{pmatrix}$, with

$$R_1 = \begin{pmatrix} \sigma_\epsilon & 0 \\ 0 & \sigma_\eta \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \qquad R_2 = \begin{pmatrix} & 0 & 0 \\ \mathbf{0}_{4\times 2} & 0 & 0 \\ & \sigma_\chi & 0 \\ & 0 & \alpha\gamma\sigma^2\sigma_\xi \end{pmatrix}.$$

If the state vector of higher-order expectations evolves according to equation (7.29), then Bayesian updating implies both that the exchange rate in period $t+1$ is conditionally normally distributed (this justifies the assumption of conditional normality) and that

$$E_{it}[Q_t(k)] = ME_{it-1}[Q_{t-1}(k)] + K\left(z_{it} - DME_{it-1}[Q_{t-1}(k)]\right),$$

where K is the Kalman gain matrix. Averaging this equation over all investors yields

$$\begin{aligned} \overline{E}_t[Q_t(k)] &= M\overline{E}_{t-1}[Q_{t-1}(k)] + K\left(DMQ_{t-1}(k) + (DN + R_2)w_t - DM\overline{E}_{t-1}[Q_{t-1}(k)]\right) \\ &= (M - KDM)\overline{E}_{t-1}[Q_{t-1}(k)] + KDMQ_{t-1}(k) + K(DN + R_2)w_t. \end{aligned} \tag{7.30}$$

Equation (7.30) implies that

$$Q_t(k) = \begin{pmatrix} q_{0t} \\ \overline{E}_t[Q_t(k-1)] \end{pmatrix} = M\begin{pmatrix} q_{0t-1} \\ \overline{E}_{t-1}[Q_{t-1}(k-1)] \end{pmatrix} + Nw_t = MQ_{t-1}(k) + Nw_t, \tag{7.31}$$

where

$$M = \begin{pmatrix} \rho_f & 0 & \mathbf{0}_{2\times 2k} \\ 0 & \rho_\nu & \\ & \mathbf{0}_{2k\times 2k+2} & \end{pmatrix} + \begin{pmatrix} & \mathbf{0}_{2\times 2k+2} \\ \mathbf{0}_{2k\times 2} & [M - KDM]_- \end{pmatrix} + \begin{pmatrix} \mathbf{0}_{2\times 2k+2} \\ [KDM]_- \end{pmatrix}, \tag{7.32}$$

$$N = \begin{pmatrix} \sigma_\zeta & 0 & \mathbf{0}_{2\times 2} \\ 0 & \sigma_\delta & \\ & [K(DN + R_2)]_- & \end{pmatrix}, \tag{7.33}$$

and $[M - KDM]_-$ is the matrix $M - KDM$ with the last two rows and columns removed and $[KDM]_-$ and $[K(DN + R_2)]_-$ are, respectively, the matrices KDM and $K(DN + R_2)$ with the last two rows removed. The Kalman gain matrix K is given by

$$K = (PD' + NR_2')(DPD' + RR')^{-1}, \tag{7.34}$$

where P satisfies the matrix Riccati equation

$$P = M\left(P - (PD' + NR_2')(DPD' + RR')^{-1}(PD' + NR_2')'\right)M' + NN'. \tag{7.35}$$

The next step is to solve for the steady-state variance of the exchange rate σ^2. In order to do this, it is necessary to compute the variance-covariance matrix

$$\hat{P} = \mathrm{Var}_{it}\begin{bmatrix} Q_{t+1}(k) \\ \xi_{t+1} \end{bmatrix} = \overline{\mathrm{Var}}_t\begin{bmatrix} Q_{t+1}(k) \\ \xi_{t+1} \end{bmatrix},$$

which depends on the steady-state dynamics of a system slightly more general than the system from equation (7.29). Note that

$$\begin{pmatrix} Q_t(k) \\ \xi_t \end{pmatrix} = \begin{pmatrix} M & \mathbf{0}_{2k+2\times 1} \\ \multicolumn{2}{c}{\mathbf{0}_{1\times 2k+3}} \end{pmatrix}\begin{pmatrix} Q_{t-1}(k) \\ \xi_{t-1} \end{pmatrix} + \begin{pmatrix} N_1 & & N_2 & \\ 0 & 0 & 0 & \sigma_\xi \end{pmatrix}\begin{pmatrix} \sigma_\zeta^{-1}\zeta_t \\ \sigma_\delta^{-1}\delta_t \\ \sigma_\chi^{-1}\chi_t \\ \sigma_\xi^{-1}\xi_t \end{pmatrix},$$

where N_1 and N_2 consist, respectively, of the first two columns and the last two columns of the matrix N from equation (7.33) above, and that

$$z_{it} = \begin{pmatrix} x_{it} \\ y_{it} \\ \bar{i}_t \\ e_t \end{pmatrix} = \begin{pmatrix} 1 & 0 & \mathbf{0}_{3\times 2k+1} \\ 0 & 1 & \\ 1 & 0 & \\ \multicolumn{2}{c}{A} & \alpha\gamma\sigma^2 \end{pmatrix}\begin{pmatrix} Q_t(k) \\ \xi_t \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} \epsilon_{it} \\ \eta_{it} \\ \chi_t \end{pmatrix}.$$

This system of equations both justifies the assumption that the conditional variance is equal for all investors i and implies that the matrix $\hat{P}$ is given by the solution to the Riccati equation

$$\hat{P} = \hat{M}\left(\hat{P} - (\hat{P}\hat{D}' + \hat{N}\hat{R}_2')(\hat{D}\hat{P}\hat{D}' + \hat{R}\hat{R}')^{-1}(\hat{P}\hat{D}' + \hat{N}\hat{R}_2')'\right)\hat{M}' + \hat{N}\hat{N}', \tag{7.36}$$

where

$$\hat{M} = \begin{pmatrix} M & \mathbf{0}_{2k+2\times 1} \\ \multicolumn{2}{c}{\mathbf{0}_{1\times 2k+3}} \end{pmatrix}, \qquad \hat{N} = \begin{pmatrix} N_1 & & N_2 & \\ 0 & 0 & 0 & \sigma_\xi \end{pmatrix}, \qquad \hat{D} = \begin{pmatrix} 1 & 0 & \mathbf{0}_{3\times 2k+1} \\ 0 & 1 & \\ 1 & 0 & \\ \multicolumn{2}{c}{A} & \alpha\gamma\sigma^2 \end{pmatrix}$$

$$\hat{R} = \begin{pmatrix} \hat{R}_1 & \hat{R}_2 \end{pmatrix}, \qquad \hat{R}_1 = \begin{pmatrix} \sigma_\epsilon & 0 \\ 0 & \sigma_\eta \\ \multicolumn{2}{c}{\mathbf{0}_{2\times 2}} \end{pmatrix}, \qquad \hat{R}_2 = \begin{pmatrix} \multicolumn{4}{c}{\mathbf{0}_{2\times 4}} \\ 0 & 0 & \sigma_\chi & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Because $e_{t+1} = AQ_{t+1}(k) + \alpha\gamma\sigma^2\xi_{t+1}$, it follows that

$$\sigma^2 = \begin{pmatrix} A & \alpha\gamma\sigma^2 \end{pmatrix}\hat{P}\begin{pmatrix} A & \alpha\gamma\sigma^2 \end{pmatrix}'. \tag{7.37}$$

I conclude that the matrices M and N and the steady-state variance σ^2 from the approximate equilibrium of Proposition 6.6 are given by the joint solution to equations (7.32), (7.33), (7.34), (7.35), (7.36), and (7.37). The fact that this approximation converges to the true steady-state equilibrium of this model is shown by Nimark (2011). □

Proof of Proposition 6.7 Suppose that the steady-state equilibrium exchange rate in period $t+1$ is normally distributed conditional on investor i's information set in period t. Suppose also that the conditional variance $\mathrm{Var}_{it}[\tilde{e}_{t+1}]$ is equal for all investors i. Lemma 6.2 then implies that the equilibrium exchange rate in period t must satisfy

$$\tilde{e}_t = \sum_{n=0}^{\infty} \alpha^{n+1} \overline{E}_t^n [f_{t+n}] + \gamma \tilde{\sigma}^2 \sum_{n=0}^{\infty} \alpha^{n+1} \overline{E}_t^n [\nu_{t+n}] + \alpha \gamma \tilde{\sigma}^2 \xi_t. \tag{7.38}$$

The exchange rate in period t is of the form

$$\tilde{e}_t = \tilde{A} \tilde{Q}_t(k) + \frac{\alpha \gamma \tilde{\sigma}^2}{1 - \alpha \rho_\nu} \nu_t + \alpha \gamma \tilde{\sigma}^2 \xi_t, \tag{7.39}$$

$$Q_t(k) = \tilde{M} \tilde{Q}_{t-1}(k) + \tilde{N} \tilde{w}_t, \tag{7.40}$$

where $k > 0$ is the level at which higher-order expectations are truncated in the model. The goal is to solve for the equilibrium conditions that characterize the matrices $\tilde{M}$ and $\tilde{N}$, the vector $\tilde{A}$, and the steady-state variance $\tilde{\sigma}^2$.

As in Proposition 6.6, the investors do not publicly observe the value of f_t in each period t, and so higher-order expectations of this interest rate parameter are part of the equilibrium exchange rate. However, unlike in Proposition 6.6, the investors do publicly observe ν_t and hence there are no higher-order expectations of current or future interventions. It follows that $\overline{E}_t^n [f_{t+n}] = h_1' (\tilde{M} \tilde{H})^n \tilde{Q}_t(k)$ for all $n \geq 1$ as before, while now $\overline{E}_t^n [\nu_{t+n}] = \rho_\nu^n \nu_t$ for all $n \geq 1$. Equation (7.38) then implies that

$$\tilde{e}_t = \sum_{n=0}^{\infty} \alpha^{n+1} h_1' (\tilde{M} \tilde{H})^n \tilde{Q}_t(k) + \frac{\alpha \gamma \tilde{\sigma}^2}{1 - \alpha \rho_\nu} \nu_t + \alpha \gamma \tilde{\sigma}^2 \xi_t,$$

so it follows by equation (7.39) that the vector $\tilde{A}$ must satisfy

$$\tilde{A} = \sum_{n=0}^{\infty} \alpha^{n+1} h_1' (\tilde{M} \tilde{H})^n.$$

Note that this equation matches equation (6.35) exactly, so that all that remains of this proof is to characterize the state transition matrices $\tilde{M}$ and $\tilde{N}$ and the steady-state variance $\tilde{\sigma}^2$.

Let $\tilde{\tilde{e}}_t = \tilde{e}_t - \frac{\alpha \gamma \tilde{\sigma}^2}{1 - \alpha \rho_\nu} \nu_t$. If the foreign central bank announces the value of ν_t publicly, the relevant observations for each investor i in each period t are given by

$$\tilde{z}_{it} = \begin{pmatrix} x_{it} \\ \bar{i}_t \\ \tilde{\tilde{e}}_t \end{pmatrix} = D \tilde{Q}_t(k) + R \begin{pmatrix} \sigma_\epsilon^{-1} \epsilon_{it} \\ \sigma_\zeta^{-1} \zeta_t \\ \sigma_\chi^{-1} \chi_t \\ \sigma_\xi^{-1} \xi_t \end{pmatrix},$$

where

$$D = \begin{pmatrix} 1 & \mathbf{0}_{2\times k} \\ 1 & \\ & \tilde{A} \end{pmatrix},$$

and $R = \begin{pmatrix} R_1 & R_2 \end{pmatrix}$, with

$$R_1 = \begin{pmatrix} \sigma_\epsilon \\ 0 \\ 0 \end{pmatrix}, \qquad R_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \sigma_\chi & 0 \\ 0 & 0 & \alpha\gamma\tilde{\sigma}^2\sigma_\xi \end{pmatrix}.$$

If the state vector of higher-order expectations evolves according to equation (7.40), then Bayesian updating implies both that the exchange rate in period $t+1$ is conditionally normally distributed (this justifies the assumption of conditional normality) and that

$$E_{it}[\tilde{Q}_t(k)] = \tilde{M}E_{it-1}[\tilde{Q}_{t-1}(k)] + K\left(\tilde{z}_{it} - D\tilde{M}E_{it-1}[\tilde{Q}_{t-1}(k)]\right),$$

where K is the Kalman gain matrix. Averaging this equation over all investors yields

$$\begin{aligned} \overline{E}_t[\tilde{Q}_t(k)] &= \tilde{M}\overline{E}_{t-1}[\tilde{Q}_{t-1}(k)] + K\left(D\tilde{M}\tilde{Q}_{t-1}(k) + (D\tilde{N} + R_2)\tilde{w}_t - D\tilde{M}\overline{E}_{t-1}[\tilde{Q}_{t-1}(k)]\right) \\ &= (\tilde{M} - KD\tilde{M})\overline{E}_{t-1}[\tilde{Q}_{t-1}(k)] + KD\tilde{M}\tilde{Q}_{t-1}(k) + K(D\tilde{N} + R_2)\tilde{w}_t. \end{aligned} \tag{7.41}$$

Equation (7.41) implies that

$$\tilde{\Pi}_t(k) = \begin{pmatrix} \tilde{q}_{0t} \\ \overline{E}_t[\tilde{Q}_t(k-1)] \end{pmatrix} = \tilde{M}\begin{pmatrix} \tilde{q}_{0t-1} \\ \overline{E}_{t-1}[\tilde{Q}_{t-1}(k-1)] \end{pmatrix} + \tilde{N}\tilde{w}_t = \tilde{M}\tilde{Q}_{t-1}(k) + \tilde{N}\tilde{w}_t, \tag{7.42}$$

where

$$\tilde{M} = \begin{pmatrix} \rho_f & \mathbf{0}_{1\times k} \\ \mathbf{0}_{k\times k+1} & \end{pmatrix} + \begin{pmatrix} & \mathbf{0}_{1\times k+1} \\ \mathbf{0}_{k\times 1} & [\tilde{M} - KD\tilde{M}]_- \end{pmatrix} + \begin{pmatrix} \mathbf{0}_{1\times k+1} \\ [KD\tilde{M}]_- \end{pmatrix}, \tag{7.43}$$

$$\tilde{N} = \begin{pmatrix} \sigma_\zeta & 0 & 0 \\ [K(D\tilde{N} + R_2)]_- & & \end{pmatrix}, \tag{7.44}$$

and $[\tilde{M} - KD\tilde{M}]_-$ is the matrix $\tilde{M} - KD\tilde{M}$ with the last row and column removed and $[KD\tilde{M}]_-$ and $[K(D\tilde{N} + R_2)]_-$ are, respectively, the matrices $KD\tilde{M}$ and $K(D\tilde{N} + R_2)$ with the last row removed. The Kalman gain matrix K is given by

$$K = (PD' + \tilde{N}R_2')(DPD' + RR')^{-1}, \tag{7.45}$$

where P satisfies the matrix Riccati equation

$$P = \tilde{M}\left(P - (PD' + \tilde{N}R_2')(DPD' + RR')^{-1}(PD' + \tilde{N}R_2')'\right)\tilde{M}' + \tilde{N}\tilde{N}'. \tag{7.46}$$

As in the proof of Proposition 6.6, the final step is to solve for the steady-state variance of the exchange rate $\tilde{\sigma}^2$. In order to do this, it is necessary to compute the variance-covariance matrix

$$\hat{P} = \mathrm{Var}_{it}\begin{bmatrix}\tilde{Q}_{t+1}(k) \\ \xi_{t+1}\end{bmatrix} = \overline{\mathrm{Var}}_t\begin{bmatrix}\tilde{Q}_{t+1}(k) \\ \xi_{t+1}\end{bmatrix},$$

which depends on the steady-state dynamics of a system slightly more general than the system from equation (7.40). Note that

$$\begin{pmatrix}\tilde{Q}_t(k) \\ \tilde{\xi}_t\end{pmatrix} = \begin{pmatrix}\tilde{M} & \mathbf{0}_{k+1\times 1} \\ \multicolumn{2}{c}{\mathbf{0}_{1\times k+2}}\end{pmatrix}\begin{pmatrix}\tilde{Q}_{t-1}(k) \\ \tilde{\xi}_{t-1}\end{pmatrix} + \begin{pmatrix}\multicolumn{2}{c}{\tilde{N}_1} & \tilde{N}_2 \\ 0 & 0 & \sigma_\xi\end{pmatrix}\begin{pmatrix}\sigma_\zeta^{-1}\zeta_t \\ \sigma_\chi^{-1}\chi_t \\ \sigma_\xi^{-1}\xi_t\end{pmatrix},$$

where $\tilde{N}_1$ and $\tilde{N}_2$ consist, respectively, of the first two columns and the last column of the matrix $\tilde{N}$ from equation (7.44) above, and that

$$\tilde{z}_{it} = \begin{pmatrix}x_{it} \\ \bar{i}_t \\ \tilde{\bar{e}}_t\end{pmatrix} = \begin{pmatrix}1 & \mathbf{0}_{2\times k+1} \\ 1 & \\ A & \alpha\gamma\tilde{\sigma}^2\end{pmatrix}\begin{pmatrix}\tilde{Q}_t(k) \\ \xi_t\end{pmatrix} + \begin{pmatrix}1 & 0 \\ 0 & 1 \\ 0 & 0\end{pmatrix}\begin{pmatrix}\epsilon_{it} \\ \chi_t\end{pmatrix}.$$

This system of equations both justifies the assumption that the conditional variance is equal for all investors i and implies that the matrix $\hat{P}$ is given by the solution to the Riccati equation

$$\hat{P} = \hat{M}\left(\hat{P} - (\hat{P}\hat{D}' + \hat{N}\hat{R}_2')(\hat{D}\hat{P}\hat{D}' + \hat{R}\hat{R}')^{-1}(\hat{P}\hat{D}' + \hat{N}\hat{R}_2')'\right)\hat{M}' + \hat{N}\hat{N}', \tag{7.47}$$

where

$$\hat{M} = \begin{pmatrix}\tilde{M} & \mathbf{0}_{k+1\times 1} \\ \multicolumn{2}{c}{\mathbf{0}_{1\times k+2}}\end{pmatrix}, \qquad \hat{N} = \begin{pmatrix}\multicolumn{2}{c}{\tilde{N}_1} & \tilde{N}_2 \\ 0 & 0 & \sigma_\xi\end{pmatrix}, \qquad \hat{D} = \begin{pmatrix}1 & \mathbf{0}_{2\times k+1} \\ 1 & \\ \tilde{A} & \alpha\gamma\tilde{\sigma}^2\end{pmatrix},$$

$$\hat{R} = \begin{pmatrix}\hat{R}_1 & \hat{R}_2\end{pmatrix}, \qquad \hat{R}_1 = \begin{pmatrix}\sigma_\epsilon \\ 0 \\ 0\end{pmatrix}, \qquad \hat{R}_2 = \begin{pmatrix}0 & 0 & 0 \\ 0 & \sigma_\chi & 0 \\ 0 & 0 & 0\end{pmatrix}.$$

Because $\tilde{e}_{t+1} = \tilde{A}\tilde{\Pi}_{t+1}(k) + \frac{\alpha\gamma\tilde{\sigma}^2}{1-\alpha\rho_\nu}\nu_t + \alpha\gamma\tilde{\sigma}^2\xi_{t+1}$, it follows that

$$\tilde{\sigma}^2 = \begin{pmatrix}\tilde{A} & \alpha\gamma\tilde{\sigma}^2\end{pmatrix}\hat{P}\begin{pmatrix}\tilde{A} & \alpha\gamma\tilde{\sigma}^2\end{pmatrix}' + \left(\frac{\alpha\gamma\tilde{\sigma}^2}{1-\alpha\rho_\nu}\right)^2\sigma_\delta^2. \tag{7.48}$$

I conclude that the matrices $\tilde{M}$ and $\tilde{N}$ and the steady-state variance $\tilde{\sigma}^2$ from the approximate equilibrium of Proposition 6.7 are given by the joint solution to equations (7.43), (7.44), (7.45), (7.46), (7.47), and (7.48). The fact that this approximation converges to the true steady-state equilibrium of this model is shown by Nimark (2011). $\square$

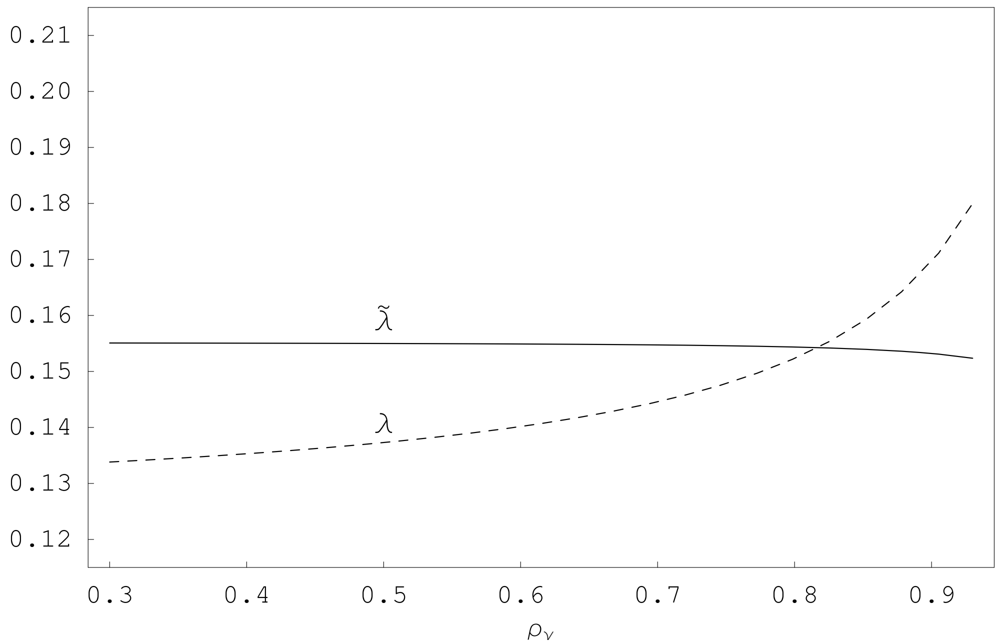

Figure 7: The value of λ (dashed line) and $\tilde{\lambda}$ (solid line) as the persistence of foreign central bank interventions ρ_ν increases. ($\sigma_\epsilon = 0.35$, $\sigma_\eta = 0.35$, $\sigma_\xi = 0.12$, $\sigma_\zeta = 0.035$, $\sigma_\delta = 0.07$, $\alpha = 0.92$, $\gamma = 5$, $\rho_f = 0.7$)

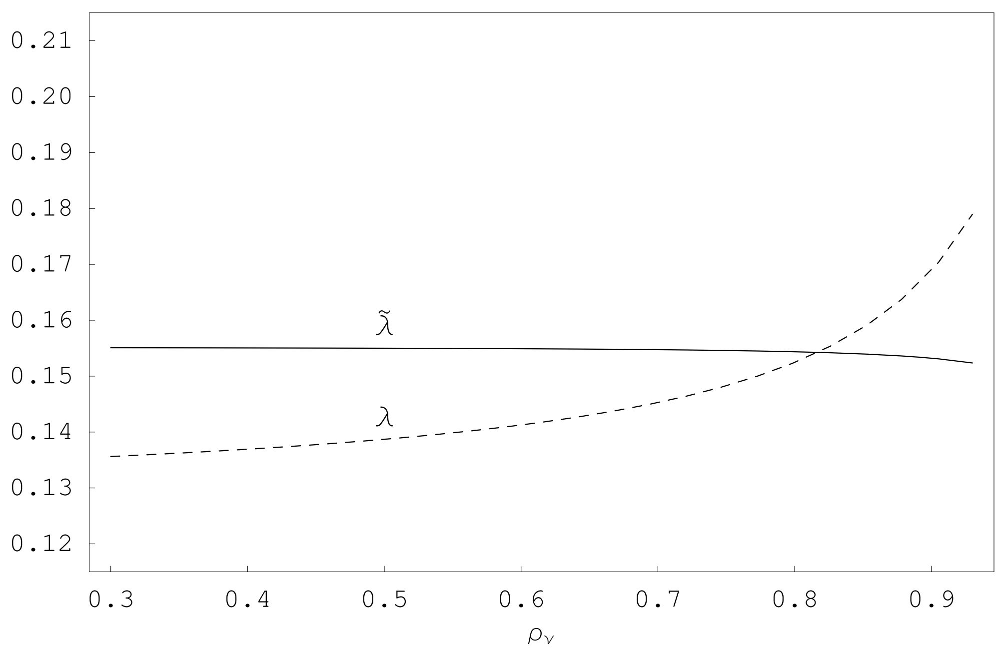

Figure 8: The value of λ (dashed line) and $\tilde{\lambda}$ (solid line) as the persistence of foreign central bank interventions ρ_ν increases. ($\sigma_\epsilon = 0.35$, $\sigma_\eta = 0.28$, $\sigma_\xi = 0.12$, $\sigma_\zeta = 0.035$, $\sigma_\delta = 0.07$, $\alpha = 0.92$, $\gamma = 5$, $\rho_f = 0.7$)

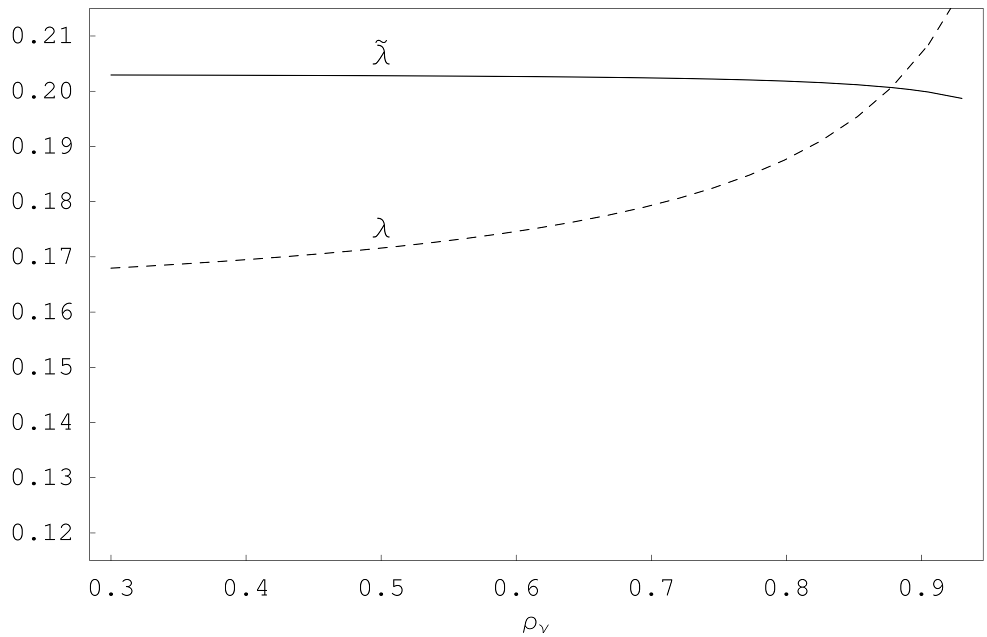

Figure 9: The value of λ (dashed line) and $\tilde{\lambda}$ (solid line) as the persistence of foreign central bank interventions ρ_ν increases. ($\sigma_\epsilon = 0.35$, $\sigma_\eta = 0.28$, $\sigma_\xi = 0.1$, $\sigma_\zeta = 0.035$, $\sigma_\delta = 0.07$, $\alpha = 0.92$, $\gamma = 5$, $\rho_f = 0.7$)

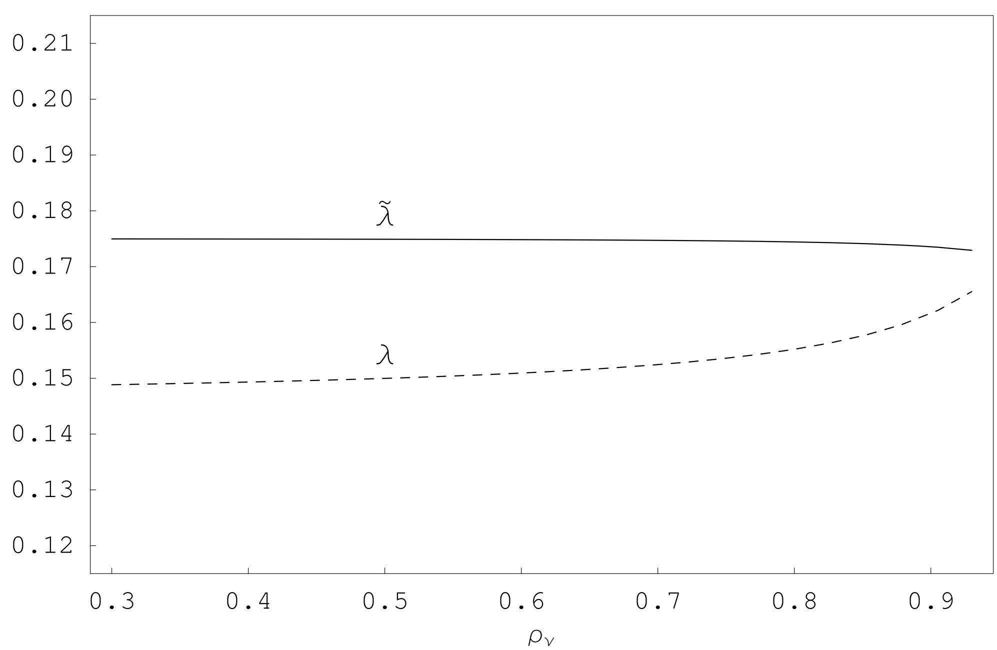

Figure 10: The value of λ (dashed line) and $\tilde{\lambda}$ (solid line) as the persistence of foreign central bank interventions ρ_ν increases. ($\sigma_\epsilon = 0.35$, $\sigma_\eta = 0.28$, $\sigma_\xi = 0.1$, $\sigma_\zeta = 0.035$, $\sigma_\delta = 0.07$, $\alpha = 0.92$, $\gamma = 5$, $\rho_f = 0.55$)

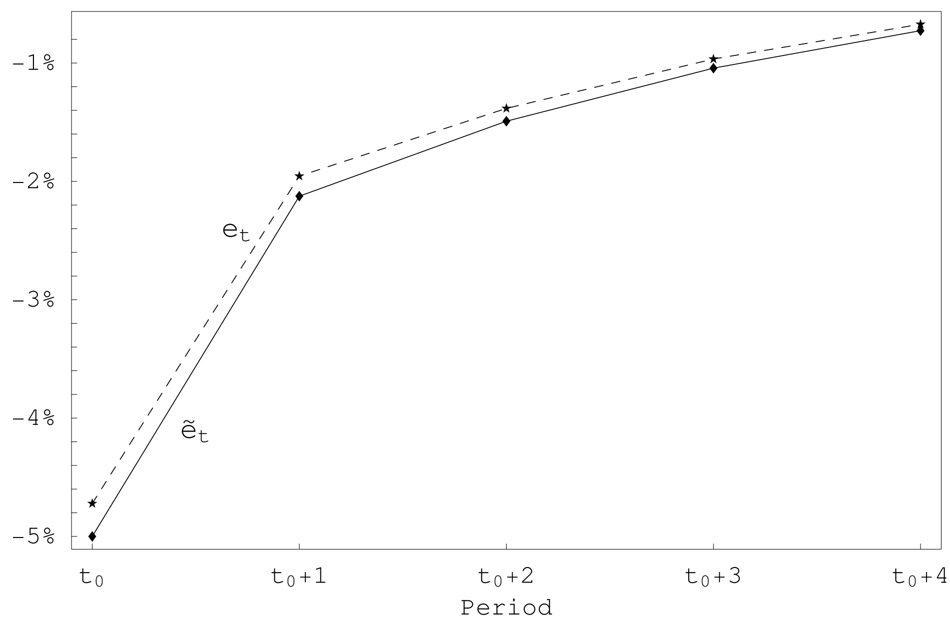

Figure 11: The response of the exchange rate with and without transparency to a shock to the noise traders' demand for peso bonds ξ_t in period t_0. ($\sigma_\epsilon = 0.35$, $\sigma_\eta = 0.35$, $\sigma_\xi = 0.1$, $\sigma_\zeta = 0.03$, $\sigma_\delta = 0.07$, $\sigma_\chi = 0.005$, $\alpha = 0.92$, $\gamma = 5$, $\rho_f = 0.7$, $\rho_\nu = 0.1$, $k = 50$)