The Rise of Big U.S. Banks and the Fall of Big European Banks: A Statistical Decomposition☆

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Abstract

Bank asset concentration has risen in the U.S. since the 1980s and declined in Europe since 2008. We decompose this rise and fall of big banks using rank-based empirical methods that characterize dynamic power law distributions into two shaping factors: the growth rates and idiosyncratic volatilities of assets for different size-ranked banks. Higher relative growth rates for the largest U.S. banks led to greater asset concentration. Idiosyncratic volatilities for U.S. banks declined, resulting in lower fundamental volatility of bank assets — the aggregate volatility due to idiosyncratic, bank-specific shocks — despite the rise in concentration since the 1990s. In contrast, the relative growth rates for the largest European banks declined and this led to the lower concentration of European bank assets. Over this same time period, the idiosyncratic volatilities and fundamental volatility of European bank assets have been stable.

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JEL: G21, C81, E58, C14

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1. Introduction

The U.S. banking sector has undergone a major transformation over the last half century. A small group of the largest banks holds more assets than ever before, a trend that accelerated after large-scale bank deregulation in the late 1990s [18, 17]. The ten largest bank-holding companies controlled about 70 percent of total bank assets by 2010. In contrast, the ten largest European commercial banks saw their share of total assets fall by one third from 2008-2016.

Using rank-based empirical methods, we explore the causes and implications of the rise of big U.S. banks and the fall of big European banks (Figure 1). Our general methods, which follow Fernholz [9], statistically decompose the stationary distribution of bank assets in terms of only two factors — the reversion rates and idiosyncratic volatilities of bank assets. The reversion rates measure the relative asset growth rates of different size-ranked banks, and capture the pace of cross-sectional mean reversion and the natural tendency of the largest institutions to grow slower than the aggregate bank population. Higher reversion rates, which mean faster cross-sectional mean reversion, imply less bank asset concentration. Idiosyncratic volatility, on the other hand, propels individual institutions away from the center of the distribution and increases asset dispersion and concentration at the top. In the banking context, idiosyncratic volatility thus has a dual role — as a source of aggregate risk [1, 22] and also as a shaping factor of concentration. Our empirical framework allows us to simultaneously investigate changes in both idiosyncratic bank asset volatility and the power law structure of the bank size distribution.

We show that both U.S. bank reversion rates and idiosyncratic asset volatilities decreased from 1986-2016 (Figures 2 and 3). This decline in reversion rates implies higher relative asset growth rates for the largest banks and explains the rise of big U.S. banks. In contrast, the fall of big European banks since 2008 is explained by a decline in the relative asset growth rates of the largest European banks. Over this same time period, the idiosyncratic volatilities of European bank assets do not meaningfully change. Our contrasting results for U.S. and European asset reversion rates are consistent with recent analyses of the diverging evolution of market power and concentration in the U.S. versus Europe [21].

A growing literature emphasizes idiosyncratic, firm-specific shocks as a potential source of aggregate volatility, especially when firm size distributions are right-skewed and follow power laws [13]. Carvalho and Gabaix [6], for example, show that "fundamental volatility" — volatility derived from microeconomic shocks alone — may be an important contributor to aggregate macroeconomic volatility. Acemoglu et al. [1] and Caballero and Simsek [5] show that firm-specific shocks are most likely to affect aggregate macroeconomic outcomes in industries with complex and opaque interlinkages. Motivated by these insights, Amiti and Weinstein [2] show that idiosyncratic granular bank supply shocks explain almost half of aggregate loan and investment fluctuations in Japan. Blank et al. [3] and Buch and Neugebauer [4] link idiosyncratic shocks at the largest European banks to changes in real GDP growth and financial stability.

We introduce the concept of heterogeneous fundamental volatility by extending the theoretical framework of Gabaix [13] to allow for idiosyncratic, bank-specific asset volatilities that vary across different size-ranked banks. Using this concept, we show that the decline in the idiosyncratic asset volatilities of the largest U.S. banks has lowered the fundamental volatility of U.S. bank assets after the mid-1990s. Crucial to this result is rank-based heterogeneity of idiosyncratic volatility. If idiosyncratic asset volatilities had not decreased at the largest U.S. banks over the past decade, then fundamental volatility would have increased. For the European banking sector, rank-based variation in idiosyncratic asset volatility is less important. Fundamental volatility of European bank assets declined since 2008, and this decline is mostly a result of the fall in European bank asset concentration over this same time period.

We also uncover a tighter link between fundamental and aggregate asset volatility for U.S. banks relative to European banks. For this result, aggregate volatility is defined as the volatility of all bank assets together. These contrasting results suggest that idiosyncratic, bank-specific shocks are an important driver of aggregate asset volatility in the U.S., while common, aggregate shocks that affect the entire banking sector such as the recent sovereign debt crisis are more important in Europe.

2. Data and Methods

In this section, we discuss our data and the rank-based econometric methods we use to analyze these data.

2.1. Bank-Level Data

We consider U.S. bank-holding companies that have to file quarterly balance sheets and income statements with the Federal Reserve.[1] These quarterly balance sheets are publicly available from the Federal Financial Institutions Examination Council (FFIEC). Since this paper focuses on the factors that shape the size distribution, the only variable we use is total institution assets, which is mnemonic BHCK2170 from regulatory form FR Y-9C.

We extract regulatory data from the so-called "call" reports. This is a repeated $N \times T$ cross-section where N is the number of banks in the cross-section and T is the quarter. Within our sample, the maximum number of bank-holding companies in a quarter is 2,338 (2005 Q2), and the minimum number in a quarter is 650 (2016 Q3). Because bank size thresholds for required reporting vary over time, the sampling of quarterly reports also varies over time. Our empirical approach requires a fixed number of ranks over time, so we size-rank all banks within reporting quarter and restrict our analysis to the largest 500 bank-holding companies each quarter. Despite the time variation in the number of reporting banks, by focusing on the 500 largest bank-holding companies we are able to cover the vast majority of U.S. bank assets throughout our 1986 Q2 - 2016 Q3 sample period.

The total assets of European commercial banks from 2008 Q1 - 2016 Q3 are obtained from S&P Global Market Intelligence. As with U.S. bank-holding companies, we do not follow a fixed panel of European banks but instead focus on a changing set of the 100 largest banks by U.S.$ assets in each quarter. Over our 2008 Q1 - 2016 Q3 sample period, the number of banks ranges from 697 (2016 Q2) to 138 (2008 Q1). We restrict our analysis to commercial banks in 23 developed European countries which include all countries on the Euro as of 2010 as well as the Czech Republic, Denmark, Iceland, Norway, Sweden, Switzerland, and the United Kingdom.

Because we follow U.S. and European banks over many years, entry and exit as well as other factors constantly change the individual banks that occupy the top ranks. For example, if one bank merges with another, then the target bank drops out of the data set while the acquiring bank remains in the data set and grows larger. Conversely, if a bank divests some of its assets and the spin-off is a new bank, then both entities will be in the data

[1] A bank-holding company is defined as a company that owns a deposit-taking financial institution (see Chapters 16 and 17 of Title 12 of the United States Code from 1841).

set going forward, with the original bank smaller than before the divestiture. Thus, we do not follow a fixed panel of banks every quarter, but instead a changing set of the largest banks in each quarter.

2.2. Rank-Based Methods

We use the rank-based empirical methods for dynamic power law distributions detailed by Fernholz [9] to characterize the distribution of U.S. and European bank assets. These methods are a multivariate extension of the single random growth process studied by Gabaix [11], and yield an asymptotic identity that describes the asset distribution according to the relationship

$$\text{bank asset concentration} \;=\; \frac{\text{idiosyncratic volatility of bank assets}}{\text{reversion rates of bank assets}}. \tag{1}$$

This statistical decomposition motivates our empirical strategy. In particular, (1) implies that any increase in bank asset concentration must be caused, in an econometric sense, by either an increase in idiosyncratic asset volatility or a decrease in reversion rates. Intuitively, higher volatility causes bank assets to become dispersed and concentrate at the top, and lower reversion rates, which imply slower reversion to the mean, cause bank assets to stay concentrated at the top.

In order to characterize the distribution of bank assets using our framework, it is necessary to consider the dynamics of bank assets by rank. Typically, bank-level empirical work estimates moments related to individual banks, such as Bank of America, J.P. Morgan, or Wells Fargo. Individual banks dynamically move through the bank size distribution. For example, an individual bank might temporarily grow faster than similar sized banks, and hence will rise in size-rank over time. If the pace of these rank crossings changes over time, this will change the bank size distribution itself. A stationary distribution is characterized by stable asset shares for each size-rank.

Figure 1 shows the changing asset shares of the top 10 and top 11-100 largest U.S. (left panel) and European (right panel) banks. Let $\theta_i(t)$ be the share of total assets held by bank i at time t, for $i = 1, \ldots, N$, and let $\theta_{(k)}(t)$ be the share of total assets held by the k-th largest bank at time t, so that $\theta_{(1)}(t) \geq \cdots \geq \theta_{(N)}(t)$ for all t. In terms of $\theta_{(k)}$, Figure 1 plots the evolution of $\theta_{(1)}(t) + \cdots + \theta_{(10)}(t)$ and $\theta_{(11)}(t) + \cdots + \theta_{(100)}(t)$ over time. This figure shows a clear rise in U.S. bank asset concentration starting in the 1990s and a fall in European bank asset concentration starting in 2008.

Idiosyncratic Volatilities $\boldsymbol{\sigma_k}$. The idiosyncratic volatilities σ_k are defined as the time-averaged volatility of the relative asset holdings of the k-th and $(k+1)$-th largest banks, for each pair of adjacent ranks $k, k+1$ in the distribution. As with the growth rates that determine the reversion rates α_k, the relative asset volatilities used to calculate σ_k can vary over time and across any bank characteristics. These different and changing volatilities are averaged over time for each pair of adjacent ranks $k, k+1$, and this yields rank-based volatilities σ_k. In the presence of both idiosyncratic, bank-specific shocks and aggregate shocks, these volatility parameters will measure only the intensity of idiosyncratic shocks since aggregate shocks that affect all banks have no impact on the relative asset holdings of adjacent banks in the distribution.

Reversion Rates $\boldsymbol{-\alpha_k}$. The reversion rates $-\alpha_k$ are defined as minus the time-averaged limit of the expected growth rate of assets for the k-th largest bank, for each rank k. The expected growth rates used to calculate α_k can vary over time and across any bank characteristics. The key insight is that by averaging these different and changing growth rates over time for each rank k, we obtain rank-based relative growth rates α_k that allow us to characterize the distribution of bank assets, as we shall describe below. The relative growth rates α_k are a measure of the rate at which bank assets revert to the mean. We refer to the $-\alpha_k$ as reversion rates, since lower values of α_k (and hence higher values of $-\alpha_k$) imply faster cross-sectional mean reversion.

The reversion rates reflect size-dependent constraints on growth such as size-dependent capital requirements for globally systemically important banks [G-SIBs as in 16, 7]. These also encompass economic mechanisms such as entry and exit, mergers and acquisitions, and regulatory and competition policy in the banking sector [18, 19], as well as the preferences, constraints, and strategic choices that drive asset growth for different sized banks [7].

Theorem 2.1. *If there is a stationary distribution of bank assets, then for* $k = 1, \dots, N$*, this distribution satisfies*

$$E\left[\log\theta_{(k)}(t) - \log\theta_{(k+1)}(t)\right] = \frac{\sigma_k^2}{-4(\alpha_1 + \cdots + \alpha_k)}. \tag{2}$$

Theorem 2.1 provides an analytic rank-by-rank characterization of the entire distribution of bank assets that matches the intuitive form of (1).[2]

[2]This theorem is from Fernholz [9], and we refer the reader there for the proof.

The theorem yields a system of $N - 1$ equations which together with the identity $\theta_{(1)} + \cdots + \theta_{(N)} = 1$ can be solved to yield the asset shares held by every single ranked bank $\theta_{(k)}$. Because the asset shares $\theta_{(k)}$ describe the cumulative distribution function (CDF) of the distribution of bank assets, we thus have a full characterization of the distribution of bank assets.

According to Theorem 2.1, two factors shape the bank size distribution. The first factor is the idiosyncratic volatilities of bank assets, σ_k, and the second factor is the reversion rates of bank assets, measured by $-\alpha_k$. Because the left-hand side of (2) corresponds to the y-axis of a log-size versus log-rank plot, we see that variation in both factors across different ranks in the distribution allows more flexibility for the shape of the bank size distribution than simpler formulations based on the equal volatilities and growth rates imposed by Gibrat's law [11, 12]. In terms of the idiosyncratic volatilities σ_k and relative growth rates α_k, Gibrat's law is equivalent to there existing some common $\sigma > 0$ and $\alpha < 0$ such that $\sigma = \sigma_1 = \cdots = \sigma_{N-1} > 0$ and $\alpha = \alpha_1 = \cdots = \alpha_{N-1} < 0$. Clearly, then, Gibrat's law is a special case of the general rank-based characterization (2).

Theorem 2.1 shows that an increase in reversion rates lowers the concentration of bank assets. An increase in idiosyncratic volatility raises the concentration of bank assets. Any change in the bank size distribution is caused by a corresponding change in at least one of these two factors that shape the distribution. By analyzing how these two shaping factors change over time we can determine the cause, in an econometric sense, of the rise of U.S. big banks and the fall of European big banks.

2.3. Estimation

We follow the econometric procedure described by Fernholz [9] to estimate the idiosyncratic volatilities σ_k and reversion rates $-\alpha_k$. However, this procedure is more complicated using data on bank assets over time because of the changes in the size distribution that have occurred in the last few decades. Figure 1 shows that both the U.S. and European bank size distributions go through a transition from one distribution to another.

In the context of our empirical approach, we model this as one-time permanent changes in idiosyncratic volatilities σ_k and reversion rates $-\alpha_k$ that occurred between 1986-2016 for the U.S. and between 2008-2016 for Europe. These permanent changes in the parameters σ_k and α_k thus lead to transitions and then new stationary distributions. It is necessary, then, to estimate the quarter in which each transition began as well as two sets of reversion rates and volatilities for each transition — one before the start, and one after it.

For each of the U.S. and Europe, we first select a quarter as the start date for the transition from one distribution to another. Next, we estimate two sets of idiosyncratic volatilities σ_k and reversion rates $-\alpha_k$ using data before and after our transition start date (this follows the procedure described above). Finally, we calculate the root mean squared error (RMSE) between the observed log asset shares $\theta_{(k)}$ and those predicted by our estimated reversion rates and volatilities according to (2). This procedure is repeated over a set of plausible start dates for the transition from one distribution to another, and then choose the transition start date that minimizes the RMSE. This data-driven transition start date is 1998 Q3 for U.S. banks and 2011 Q4 for European banks (see Figure 7). This procedure of estimating two sets of parameters and then comparing predicted and actual asset shares both before and after the transition start date does not explicitly capture the transition from one distribution to the other. It has the advantage of simplicity and tractability.

3. U.S. and European Bank Size Dynamics

The intuitive version of our statistical decomposition (1) motivates our empirical strategy. By estimating idiosyncratic volatilities σ_k and reversion rates $-\alpha_k$ for U.S. and European banks, we can examine how these two shaping factors changed over time. According to Theorem 2.1, this analysis offers an econometric explanation of the changing bank size distributions observed in Figure 1.

3.1. Idiosyncratic Volatilities

In the left panel of Figure 2, we plot the estimated standard deviations of the idiosyncratic volatilities of asset holdings for the 500 largest U.S. banks from 1986 Q2 - 1998 Q3 and 1998 Q4 - 2016 Q3. This figure shows that the idiosyncratic asset volatilities for banks decreased after 1998 Q3, with the largest decreases occurring for medium-sized banks. In the Online Appendix, we extend the results of Fernholz and Koch [10] and report the statistical significance of these changes. Consistent with Figure 2, the significance of the changes in σ_k is in fact largest for medium-sized banks (see the left panel of Figure 14).

In contrast, the right panel of Figure 2 shows that the standard deviations of the idiosyncratic volatilities of asset holdings for the 100 largest European banks are approximately unchanged from 2008 Q1 - 2011 Q4 to 2012 Q1 - 2016 Q3. According to the decomposition (1), the relatively constant idiosyncratic volatilities for European banks suggest that the relative growth rate of assets for the largest European banks went down from 2008-2016. We confirm this in Section 3.2.

In Figure 4, we plot five-quarter rolling window estimates of U.S. (left panel) and European (right panel) idiosyncratic bank volatilities for various size-ranked subsets.[3] This figure demonstrates that, with the exception of a spike around the financial crisis of 2008, U.S. bank asset volatilities have gradually declined since the late 1990s. For the top-ranked, largest banks, the figure shows that the lowest idiosyncratic volatilities on record have occurred in the last few years, while the asset volatilities of all U.S. banks have been remarkably low and stable by historical standards since the global financial crisis. The right panel of Figure 4 shows that, despite the European sovereign debt crisis, the idiosyncratic volatility at European banks was roughly constant and similar to the level for U.S. banks since 2009.

3.2. Reversion Rates

According to (2) from Theorem 2.1, the observed rise of big U.S. banks shown in the left panel of Figure 1 must be caused by either an increase in idiosyncratic volatilities σ_k, a decrease in reversion rates $-\alpha_k$, or both. Given the observed decrease in idiosyncratic asset volatilities of U.S. banks (σ_k) from Figure 2, then, it must be that cross-sectional mean reversion ($-\alpha_k$) decreased in 1998 Q4 - 2016 Q3 relative to 1986 Q2 - 1998 Q3. Similarly, the fall of European big banks shown in the right panel of Figure 1 must be caused by either a decrease in idiosyncratic volatilities σ_k, an increase in reversion rates $-\alpha_k$, or both.

Figure 3 substantiates both predictions. The left panel shows a decline in mean reversion of U.S. bank assets from 1986 Q2 - 1998 Q3 to 1998 Q4 - 2016 Q3. This fall more than offsets the fall in idiosyncratic volatility and led to the rise in U.S. bank asset concentration. Because the parameters α_k measure relative asset growth rates for different ranked banks, Figure 3

[3] In contrast to the volatilities shown in Figure 2, the rolling window estimates in Figure 4 are not smoothed so as to best match the bank size distributions in different time periods.

shows that the asset growth rates of the largest U.S. banks rose by more than 0.5 percentage points relative to all banks after 1998 Q3. Big U.S. banks now stay bigger for longer. The significant change in regulation for the largest U.S. banks in the 1990s is one explanation for this decline in mean reversion rates. Both the repeal of Glass-Steagall through the Gramm-Leach-Bliley Act [20], which had previously separated commercial and investment banking, and the removal of inter-state branching restrictions [18, 19, 8] imply relatively faster asset growth for the largest banks and hence less mean reversion.[4]

In contrast to the reduced pivot around the midpoint shown in the left panel of Figure 3, the right panel shows a relatively uniform rise in mean reversion of European bank assets from 2008 Q1 - 2011 Q4 to 2012 Q1 - 2016 Q3. This decrease in the relative asset growth rates of larger European banks led to the fall in European bank asset concentration.[5] The contrasting changes in U.S. and European asset reversion rates shown in Figure 3 highlight the diverging evolution of regulation and market power in the U.S. versus Europe [21].

3.3. Goodness of Fit

We can see from Figures 2 and 3 that the shaping parameters σ_k and α_k vary across different ranked banks in both the U.S. and Europe. Such variation in growth rates and idiosyncratic volatilities across different ranks is inconsistent with Gibrat's law, the special case of our general approach discussed in Section 2. In this sense, our rank-based framework generalizes previous studies based on the equal volatilities and growth rates imposed by Gibrat's law in a way that allows us to better match the empirical bank size distribution.

The left panel of Figure 5 shows the average share of total assets held by different ranked U.S. banks from 1986 Q2 - 1998 Q3 together with the shares predicted for these banks using equation (2) from Theorem 2.1 estimated over this same time period.[6] The right panel of Figure 5 shows these same

[4]Furthermore, Table 1 in the Online Appendix shows that the Gramm-Leach-Bliley Act coincides with statistically significant increases in the log-log slope of bank size versus rank.

[5]Figure 14 in the Online Appendix reports the statistical significance of the decline in the parameters α_k and reveals that the most significant declines occur for medium-sized European banks, consistent with the magnitudes observed in Figure 3.

[6]The figure displays asset shares as a function of rank, using log scales for both axes.

quantities for different ranked U.S. banks from 1998 Q4 - 2016 Q3. These two panels are constructed using the cross-sectional idiosyncratic volatility σ_k and mean-reversion $-\alpha_k$ parameters from the left panels of Figures 2 and 3. Together with (2), these parameter values yield stationary distribution values for each rank asset share $\theta_{(k)}$. As the two figures demonstrate, equation (2) estimated over these two different time periods is able to accurately match the observed U.S. bank size distribution. Furthermore, the predicted shares generate an increased concentration in bank assets for the 1998 Q4 - 2016 Q3 time period.

Similarly, Figure 6 shows the average and predicted shares of total assets held by different ranked European banks from 2008 Q1 - 2011 Q4 and 2012 Q1 - 2016 Q3. As with the U.S. data, the decomposition (2) is able to approximately match the observed European bank size distributions before and after transition despite a relatively small sample of 33 quarters. The predicted shares also generate a decreased concentration of European bank assets for the 2012 Q1 - 2016 Q3 time period, consistent with Figure 1.

The solid red line in the left panel of Figure 7 reports the root mean squared error (RMSE) between the predicted and observed log U.S. bank asset shares before and after the transition from small to big banks using different transition start dates (right axis). As mentioned earlier, the minimum RMSE occurs with a 1998 Q3 transition start date, and thus we use this date for our analysis in this section.[7] Similarly, the right panel of Figure 7 shows that the minimum RMSE between predicted and observed log European bank asset shares occurs with a transition start date of 2011 Q4.

In the Online Appendix, we also consider the estimated log-log slope of bank assets versus rank for the top 100 largest U.S. banks and the top 40 largest European banks using the well-known Hill estimator [15] and the more recent "Rank – 1/2" OLS estimator [14]. These estimates are shown in Figure 11. In order to examine the consistency of our RMSE analysis using our novel dynamic power law methods, we report in Figure 7 the t-statistics of Zivot-Andrews breakpoint tests using the Hill and Rank - 1/2 OLS estimates from Figure 11. For the U.S., both t-statistics generate large and statistically significant values on dates close to the 1998 Q3 date chosen

As discussed in Gabaix [12], if asset shares follow a Pareto distribution, then such a figure will appear as a straight line.

[7] Note that transition start dates outside of the range in Figure 7 — before 1996 Q2 or after 2001 Q2 — generate substantially larger RMSEs.

by our RMSE analysis. Similarly, the right panel of Figure 7 demonstrates the statistical significance of Zivot-Andrews t-statistics on dates close to the 2011 Q4 date chosen by our RMSE analysis for European banks.

4. Fundamental Volatility

Our estimates of the idiosyncratic asset volatilities from Section 3 together with the changing share of assets held by the largest banks enable us to examine the dynamics of fundamental volatility in both the U.S. and European banking sectors. To clarify our contribution, we benchmark our measure of fundamental volatility, which allows for rank-based heterogeneity in idiosyncratic volatility, to standard measures of fundamental volatility that impose uniform volatility across all size-ranks. Finally, we compare fundamental volatility in the banking sector to the aggregate volatility of bank assets.

4.1. A Rank-Based Approach

Let $a_i(t)$ denote the assets held by bank i at time t, and $a_{(k)}(t)$ denote the assets held by the k-th largest bank at time t. We follow Gabaix [13] and Carvalho and Gabaix [6] and define fundamental volatility, or the granular residual, as the volatility of total bank assets $a(t) = a_1(t) + \cdots + a_N(t)$ that results from idiosyncratic, bank-specific shocks alone.[8] In order to simplify our analysis, we assume that shocks to bank assets are either aggregate — affecting the assets of all banks in the same way — or idiosyncratic — affecting the assets of only one individual bank.

We use the rank-based continuous-time framework described in Section 2. In the context of that framework, the assumption that banks face idiosyncratic and aggregate shocks implies that, for all $i = 1, \ldots, N$, bank asset dynamics from (1) of Fernholz [9] simplify to

$$d \log a_i(t) = \mu_i(t)\, dt + \delta_i(t)\, dB_i(t) + \delta(t)\, dB_M(t), \tag{3}$$

where μ_i, δ_i, and δ are measurable and adapted processes that are otherwise general and unrestricted, and $\mathbf{B}(t) = (B_1(t), \ldots, B_M(t))$ is an M-dimensional Brownian motion with $M > N$. In the simplified bank asset dynamics (3),

[8] These definitions imply that $a_{(1)}(t) \geq \cdots \geq a_{(N)}(t)$, and that $\theta_{(k)}(t) = a_{(k)}(t)/a(t)$, for all $k = 1, \ldots, N$.

each Brownian motion B_i captures idiosyncratic shocks that affect the assets of bank i alone while the common Brownian motion B_M captures aggregate shocks that affect the assets of all banks. If we apply Itô's Lemma to (3), then we have

$$\frac{da_i(t)}{a_i(t)} = \left(\mu_i(t) + \frac{\delta_i^2(t) + \delta^2(t)}{2}\right) dt + \delta_i(t)\, dB_i(t) + \delta(t)\, dB_M(t), \quad (4)$$

for all $i = 1, \ldots, N$.

If we add up the asset changes da_i characterized in (4) for all banks $i = 1, \ldots, N$, then we have that

$$da(t) = \sum_{i=1}^{N} \left(\mu_i(t) + \frac{\delta_i^2(t) + \delta^2(t)}{2}\right) a_i(t)\, dt + \delta(t) a(t)\, dB_M(t) + \sum_{i=1}^{N} \delta_i(t) a_i(t)\, dB_i(t), \quad (5)$$

which yields

$$\begin{aligned} \frac{da(t)}{a(t)} &= \sum_{i=1}^{N} \left(\mu_i(t) + \frac{\delta_i^2(t) + \delta^2(t)}{2}\right) \frac{a_i(t)}{a(t)}\, dt + \delta(t)\, dB_M(t) + \sum_{i=1}^{N} \delta_i(t) \frac{a_i(t)}{a(t)}\, dB_i(t), \\ &= \sum_{i=1}^{N} \left(\mu_i(t) + \frac{\delta_i^2(t) + \delta^2(t)}{2}\right) \theta_i(t)\, dt + \delta(t)\, dB_M(t) + \sum_{i=1}^{N} \delta_i(t)\theta_i(t)\, dB_i(t). \end{aligned} \quad (6)$$

The first term of (6) measures the expected change in the value of total bank assets $a(t) = a_1(t) + \cdots + a_N(t)$. The second term measures the volatility of total assets caused by aggregate shocks. These first two terms do not contribute to fundamental volatility. The third term measures the volatility of total bank assets resulting from idiosyncratic, bank-specific shocks alone, which is the definition of fundamental volatility. Indeed, (6) is a continuous-time version of the characterization (3) from Gabaix [13]. Crucial to our analysis of fundamental volatility in the banking sector, however, is rank-based variation in both the asset shares θ_i and the idiosyncratic volatilities δ_i from (6). Figures 1 and 4, for example, illustrate that both the asset shares and the idiosyncratic asset volatilities of different size-ranked U.S. banks have changed over time. Our rank-based framework is uniquely suited to account for static rank-based variation in asset shares and idiosyncratic volatilities as well as changes in the structure of that variation over time.

In order to account for rank-based variation in entity-specific bank asset shares, θ_i, and entity-specific idiosyncratic asset volatilities, δ_i, we rewrite (6) in terms of rank-based asset shares, $\theta_{(k)}$, and rank-based idiosyncratic volatilities, $\delta_{(k)}$. This yields

$$\frac{da(t)}{a(t)} = \sum_{k=1}^{N} \delta_{(k)}(t)\theta_{(k)}(t)\, dB_{(k)}(t) \; + \; \begin{array}{c}\text{other terms not affecting} \\ \text{fundamental volatility}\end{array}, \tag{7}$$

which provides a rank-based version of the characterization of fundamental volatility (3) from Gabaix [13]. To our knowledge, (7) is the first granular, rank-based characterization of fundamental volatility.

An interesting implication of (7) is that fundamental volatility is determined by both the magnitude, $\delta_{(k)}$, and the weighting, $\theta_{(k)}$, of bank-specific shocks. At the same time, idiosyncratic, bank-specific shocks are also a shaping force of the bank size distribution, as described by (2) from Theorem 2.1. In particular, the rank-based parameters σ_k^2 from (2) measure the time-averaged value of $\delta_{(k)}$. Thus, idiosyncratic shocks have both a *direct* effect on fundamental volatility via their magnitude as well as an *indirect* effect via their impact on the weighting. Our novel rank-based methods allow us to characterize this dual role for idiosyncratic volatility as a source of fundamental volatility.

4.2. Fundamental Volatility and Rank-Based Heterogeneity

According to Figure 1, the share of assets held by the largest U.S. banks increased after the 1990s. Figure 4 shows that the idiosyncratic asset volatilities of the largest U.S. banks declined over this same time period. According to (7), these two changes imply opposite effects on the evolution of fundamental volatility in the U.S. banking sector over time. The rise in asset shares in the left panel of Figure 1 is equivalent to a rise in $\theta_{(k)}$ at the highest ranks k, while the decline in idiosyncratic volatilities in the left panel of Figure 4 is equivalent to a decline in $\delta_{(k)}$ at those same high ranks. Put differently, the weighting of bank-specific shocks for the biggest U.S. banks rose at the same time that the magnitude of those bank-specific shocks fell.

In order to determine which of the opposing effects uncovered by Figures 1 and 4 dominates, we use the rank-based estimation outlined in Section 2 together with the rank-based characterization of fundamental volatility (7). More specifically, we use five-quarter rolling window estimates of the parameters σ_k to estimate time-averaged values of $\delta_{(k)}$ from (7).[9] These rolling window estimates of the idiosyncratic volatilities are unsmoothed across different ranked banks, as in Figure 4. We then multiply these rank-based, five-quarter rolling window estimates of $\delta_{(k)}$ times the rank-based, single-quarter weights $\theta_{(k)}$ as in (7) to obtain estimates of fundamental volatility.

The left panel of Figure 8 plots the fundamental volatility of total U.S. bank assets over the full 1986 Q2 - 2016 Q3 time sample using the rank-based estimation described above. Other than a large and dramatic rise in fundamental volatility around the financial crisis of 2008, the figure illustrates that U.S. fundamental volatility has been gradually declining since the late 1990s, with some of the lowest levels on record occurring in the last few years. This general pattern aligns with the dynamics of the rank-based idiosyncratic asset volatilities shown in the left panel of Figure 4.

Upon careful examination, we see that the changing asset volatilities of the largest U.S. banks in the left panel of Figure 4 most closely match the changes in U.S. fundamental volatility in the left panel of Figure 8. This match is especially pronounced after the rise of big banks in the late 1990s. The characterization of fundamental volatility (7) predicts exactly this. According to (7), the idiosyncratic volatilities of the largest banks, $\delta_{(k)}$, will increasingly dominate fundamental volatility as the weight, $\theta_{(k)}$, of those banks rises. Of course, the left panel of Figure 1 shows that these weights rose during the 1990s for U.S. banks. In this way, the tight alignment after the 1990s between the volatilities of the largest U.S. banks in Figure 4 and U.S. fundamental volatility in Figure 9 is exactly what is expected.

In contrast to the U.S., European banks saw little change in idiosyncratic asset volatilities from 2008-2016, as seen in the right panels of Figures 2 and 4. During this same time period, however, Figure 1 shows a clear decline in the share of assets held by the largest European banks. According to (7), then, fundamental volatility in the European banking sector should also decrease. We confirm this prediction in the right panel of Figure 8, which demonstrates that the fundamental volatility of European bank assets fell by more than 50% from 2008-2016.

[9] Recall from Section 2 that the parameters σ_k measure the idiosyncratic asset volatilities of both the k-th and $(k+1)$-th largest banks together. In order to obtain values that correspond to idiosyncratic asset volatilities for a single ranked bank, then, it is necessary to adjust the estimates of σ_k reported in Figures 2 and 4. Our estimates of $\delta_{(k)}$ are adjusted in this way.

How important is rank-based heterogeneity for the dynamics of fundamental volatility? Figure 8 plots the fundamental volatility of bank assets when imposing Gibrat's law, which implies a common idiosyncratic volatility σ_k for all banks. This *homogeneous* fundamental volatility is plotted alongside our novel *heterogeneous* fundamental volatility that allows for rank-based variation in idiosyncratic volatilities in extension of the framework of Gabaix [13]. In the case of Europe, rank-based variation in idiosyncratic volatilities does not substantially alter the calculated values of fundamental volatility. This is apparent given the close similarity between heterogeneous (solid black line) and homogeneous (dashed red line) fundamental volatility in the right panel of Figure 8. Both measures show a decline from 2008-2016. This is not surprising, since the right panels of Figures 2 and 4 show little variation in idiosyncratic asset volatilities across different ranked European banks.

In the case of the U.S. banking sector, however, rank-based heterogeneity is crucial. This is evident from the different trajectories of heterogeneous (solid black line) and homogeneous (dashed red line) fundamental volatility in the left panel of Figure 8. If we impose a common idiosyncratic volatility across all U.S. banks, then the rising concentration of U.S. bank assets of Figure 1 dominates the calculation of fundamental volatility and leads to a substantial measured rise from 1986-2016. Our rank-based methods reveal that this measured rise is inaccurate, however, and that the true decline in U.S. fundamental volatility is a direct consequence of the decline in the idiosyncratic asset volatilities of the largest U.S. banks.

The heterogeneous and homogeneous measures of U.S. fundamental volatility also diverge markedly around the financial crisis of 2008. According to the left panel of Figure 8, there was a substantial rise in fundamental volatility during the crisis that is absent when imposing a common idiosyncratic volatility. This divergence emerges naturally from our framework, since the spike in fundamental volatility around 2008 was driven by a spike in idiosyncratic volatility at only the very largest U.S. banks, as shown in Figure 4. If we impose a common volatility and thus average this rise in volatility at the largest banks across all ranks, then the result is little change in measured fundamental volatility as shown by the relatively stable homogeneous fundamental volatility (red dashed line) in the left panel of Figure 8. The standard approach thus implausibly suggests that the financial crisis of 2008 — which emanated from the largest U.S. banks — did not meaningfully impact fundamental volatility. Therefore, for both the short- and long-run behavior of U.S. fundamental volatility, the departure from the strict form of Gibrat's law of our rank-based approach provides new insights that are not possible using more established methods.

4.3. Fundamental and Aggregate Volatility

It is useful to investigate the relationship between the fundamental volatility of bank assets and the aggregate volatility of bank assets. In Figure 9, we plot the fundamental volatility of the U.S. and European banking sectors from Figure 8 together with the aggregate asset volatility of the 500 largest U.S. banks (left panel) and the 100 largest European banks (right panel). To maintain consistency with our measure of fundamental volatility, we report aggregate volatility as the annualized standard deviation of log asset growth for U.S. and European banks.[10]

Figure 9 reveals a tighter link between fundamental and aggregate asset volatility for U.S. banks relative to European banks. Not only is the level of fundamental asset volatility closer to the level of aggregate asset volatility in the U.S. than in Europe, but Figure 9 also demonstrates that the dynamics of fundamental and aggregate volatility are more closely aligned in the U.S. than in Europe. These contrasting results suggest that idiosyncratic, bank-specific shocks are a more important driver of aggregate asset movements in the U.S. Figure 9 shows that the rise in aggregate volatility around the financial crisis of 2008 is primarily the result of a corresponding rise in fundamental volatility during this period. Furthermore, as we saw in Figures 4 and 8, this jump in U.S. fundamental volatility is driven by a jump in the magnitude of idiosyncratic shocks at the largest U.S. banks.

Our results suggest that common, aggregate shocks that affect the entire banking sector are a more important source of aggregate asset movements in Europe. This can be seen plainly by the large rise in aggregate asset volatility around the sovereign debt crisis of the early 2010s in the right panel of Figure 9. Unlike the simultaneous spike in U.S. fundamental and aggregate volatility in 2008, the rise in aggregate asset volatility in Europe in the early 2010s is not accompanied by a comparable change in the fundamental volatility of the European banking sector. This means that aggregate shocks that were common to the 100 largest European banks were the driver of this rise in asset volatility during the sovereign debt crisis.

[10] Because fundamental and aggregate volatility are measured in terms of log asset growth rather than the percentage change in assets, the sum of idiosyncratic and common shocks is not exactly equal to aggregate volatility.

The decline in fundamental volatility of the U.S. banking sector implies that one important source of macroeconomic and financial instability was historically low in 2016. Despite the tight link between fundamental and aggregate volatility in the U.S., there is always the possibility of a common, systemic shock to the banking sector, as occurred in Europe during the sovereign debt crisis. Furthermore, given that bank mergers and rising asset concentration can in fact increase systemic risk [23], the rise of big banks helps rationalize U.S. policymakers' recent focus on systemic risk.

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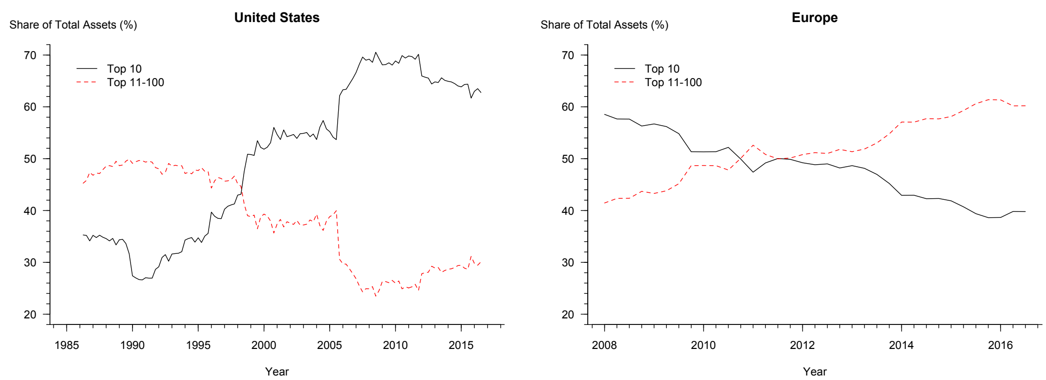

Figure 1: Shares of total assets held by the largest U.S. banks, 1986-2016 (left panel), and the largest European banks, 2008-2016 (right panel).

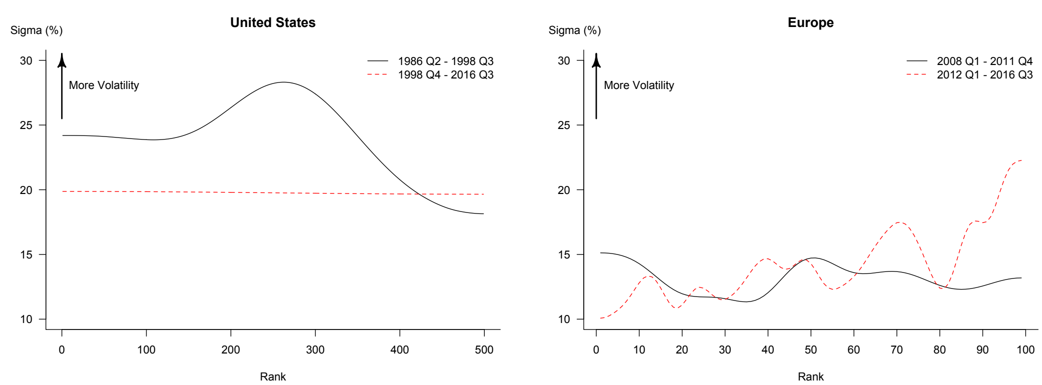

Figure 2: Standard deviations of idiosyncratic asset volatilities (σ_k) for different ranked U.S. (left panel) and European banks (right panel).

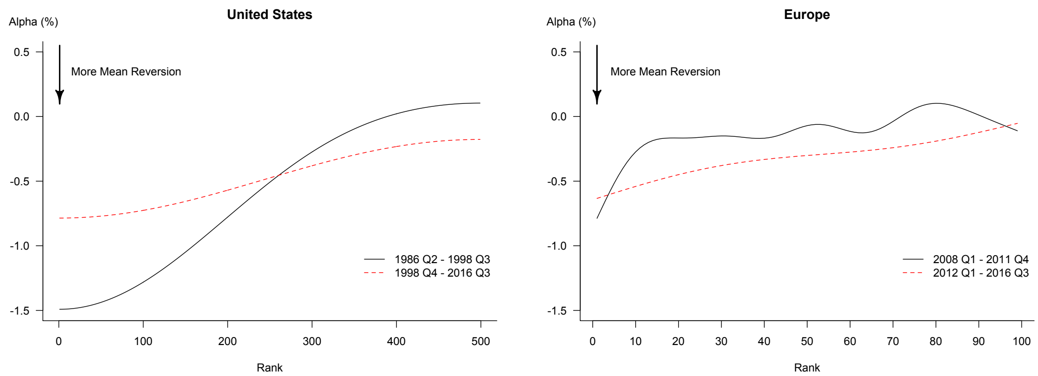

Figure 3: Minus the reversion rates (α_k) for different ranked U.S. (left panel) and European banks (right panel).

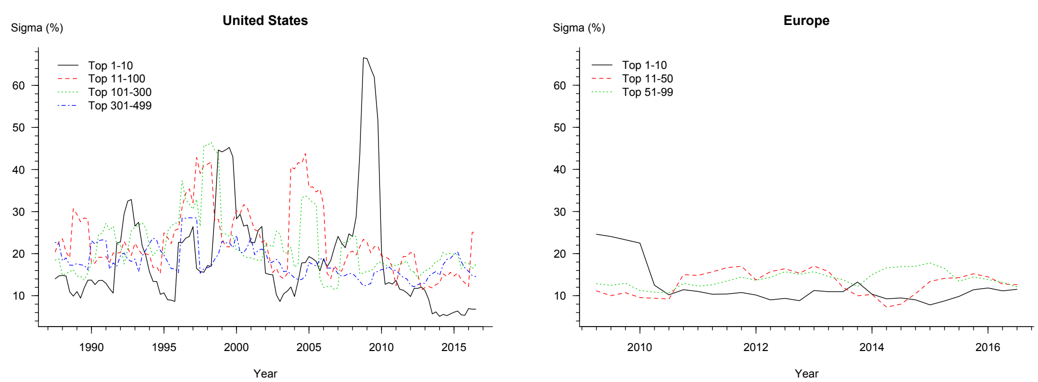

Figure 4: Five-quarter rolling window estimates of standard deviations of idiosyncratic asset volatilities (σ_k) for different ranked U.S. banks, 1986-2016 (left panel), and different ranked European banks, 2008-2016 (right panel).

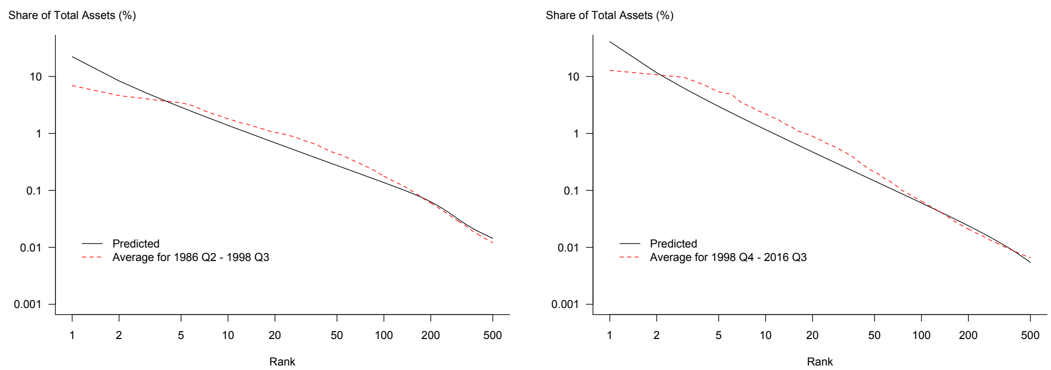

Figure 5: Shares of total assets held by the 500 largest U.S. banks, 1986 Q2 - 1998 Q3 (left panel) and 1998 Q4 - 2016 Q3 (right panel), as compared to the predicted shares.

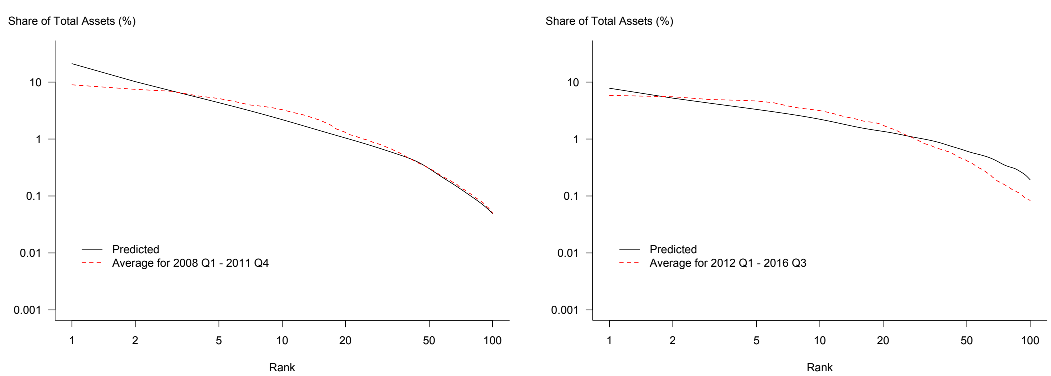

Figure 6: Shares of total assets held by the 100 largest European banks, 2008 Q1 - 2011 Q4 (left panel) and 2012 Q1 - 2016 Q3 (right panel), as compared to the predicted shares.

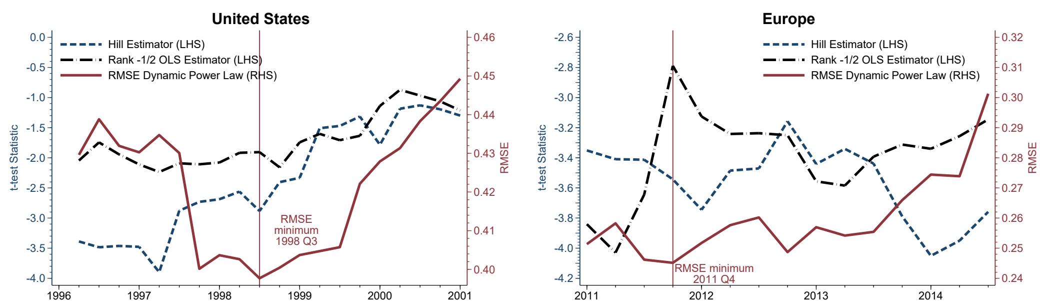

Figure 7: T-test statistics for Zivot-Andrews breakpoint tests using Hill and Rank - 1/2 OLS estimators of slope of log-log plot of bank size versus rank, and root mean squared errors between predicted and observed log bank asset shares for different dynamic power law transition start dates for U.S. (left panel) and European banks (right panel).

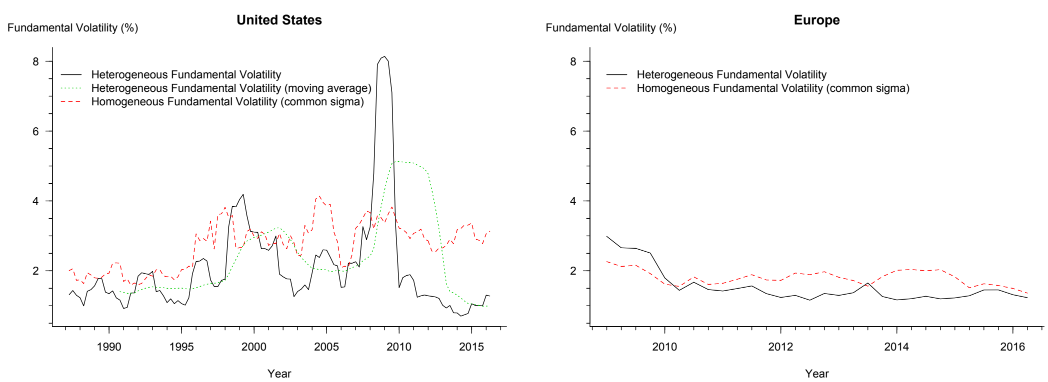

Figure 8: Heterogeneous and homogeneous fundamental volatility for the U.S. banking sector, 1986-2016 (left panel), and the European banking sector, 2008-2016 (right panel).

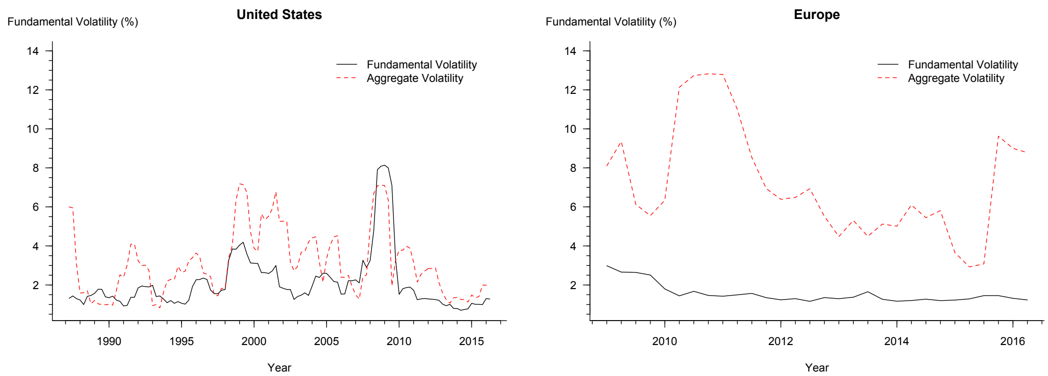

Figure 9: Fundamental volatility and aggregate volatility of the U.S. banking sector, 1986-2016 (left panel), and the European banking sector, 2008-2016 (right panel).

Online Appendix: "The Rise of Big U.S. Banks and the Fall of Big European Banks: A Statistical Decomposition" (Ricardo T. Fernholz and Christoffer Koch)

In this online appendix, we present alternative measures of the changing concentration of U.S. and European bank assets. We also present results regarding the statistical significance of the changes in idiosyncratic volatility for U.S. banks before and after the 1990s as well as the rise in reversion rates for European banks before and after 2011. Finally, we examine the robustness of our empirical approach applied to U.S. banks.

Alternate Measures of Bank Asset Concentration

Figure 10 plots the changing Herfindahl index of asset concentration — equal to the squared sum of asset shares $\theta^2_{(1)}(t) + \cdots + \theta^2_{(N)}(t)$ — for both U.S. bank-holding companies from 1986-2016 and European commercial banks from 2008-2016. A higher Herfindahl index implies greater bank asset concentration. Thus, the right panel of Figure 10 shows rising asset concentration for U.S. banks while the left panel shows falling asset concentration for European banks, consistent with Figure 1.

In Figure 11, we plot minus the estimated log-log slope of assets versus rank for the top 100 largest U.S. banks and the top 40 largest European banks in each quarter of our sample periods. Each panel of this figure plots two sets of estimates, one obtained using the well-known Hill estimator [15] and the other obtained using the more recent "Rank $-$ 1/2" OLS estimator [14]. Figure 11 concurs with Figures 1 and 10 since a more negative slope corresponds to a more concentrated asset distribution.

The estimates of the log-log slopes of assets versus rank in Figure 11 are equal to the inverse of the Pareto exponents of the U.S. and European bank size distributions. Accordingly, a smaller Pareto exponent (and hence a more negative slope) corresponds to a more concentrated distribution. Furthermore, a Pareto exponent less than one, which is equivalent to a log-log slope steeper than minus one, implies that the mean of the bank size distribution is infinite. One implication of Figure 11, then, is that the mean of the distribution of U.S. bank assets grew unbounded during the 1990s.

Finally, Table 1 reports the results of time-series regressions of the Hill and Rank - 1/2 OLS estimates of the slope of the log-log plot of U.S. bank size versus rank shown in the left panel of Figure 11. These regressions include a dummy for those quarters in which the Gramm-Leach-Bliley Act, which repealed the Glass-Steagall Act, was in effect. According to this table, the repeal of Glass-Steagall is associated with a statistically significant rise in the concentration of U.S. bank assets.

Confidence Intervals and Statistical Significance

It is not possible to generate confidence intervals and p-values using classical techniques in this setting because the empirical distributions of the idiosyncratic volatilities σ_k and reversion rates $-\alpha_k$ are unknown. However, using bootstrap resampling, it is possible to generate confidence intervals and determine the statistical significance of our results in Figures 2 and 3.

We focus first on the changes in idiosyncratic volatilities for U.S. banks shown in the left panel of Figure 2. For each each of the 1986 Q2 - 1998 Q3 and 1998 Q4 - 2016 Q3 time periods, we randomly choose pairs of consecutive quarters with replacement until we have a random sample equal in length to the original data sample. This bootstrap resampling procedure is repeated 10,000 times, thus generating 20,000 bootstrap random samples — 10,000 before 1998 Q3, and 10,000 after. Using these random samples, we generate two sets of 10,000 estimates of the idiosyncratic volatilities σ_k following the procedure of Fernholz [9], and then construct confidence intervals for each time period based on these two sets of estimates.

In Figure 12, we report point estimates and 95% confidence intervals for the idiosyncratic volatilities σ_k for different ranked U.S. bank-holding companies. These estimates are generated using the bootstrap resample procedure described above, and cover the same two time periods as in the left panel of Figure 2. Figure 12 shows that the average σ_k for the top 350 largest banks in each time period is outside of the other time period's 95% confidence interval. This result suggests that the difference between these estimates is statistically significant.

Fortunately, questions of statistical significance are easily addressed using this same method of bootstrap resampling. The left panel of Figure 14 shows the probability that the idiosyncratic volatilities σ_k for different ranked U.S. banks from 1986 Q2 - 1998 Q3 are less than or equal to the σ_k from 1998 Q4 - 2016 Q3. Like the confidence intervals displayed in Figure 12, these probabilities are based on the results of 10,000 bootstrap resample estimates of the idiosyncratic volatilities σ_k. More specifically, these probabilities are generated by examining the number of resampled data sets in which the estimated σ_k from 1986 Q2 - 1998 Q3 is less than or equal to the estimated

σ_k from 1998 Q4 - 2016 Q3.[11] This procedure is repeated for all 500 ranks in the size distribution of banks.

The computed probabilities shown in the left panel of Figure 14 are essentially sets of p-values for the hypothesis of no decrease in U.S. bank idiosyncratic asset volatilities after 1998 Q3. As we see from the figure, then, one of the most important results discussed in Section 3 — the fall in the idiosyncratic asset volatilities of the largest banks after 1998 Q3 — is statistically significant at the 5% level. This decline in idiosyncratic volatility is also statistically significant at the 1% level for some middle-sized banks.

In Section 3, we showed that the fall of European big banks after 2011 is a consequence of higher reversion rates. Accordingly, we wish to use the same bootstrap resampling procedure to investigate the statistical significance of the changes in European bank asset reversion rates shown in the right panel of Figure 3. Following the same procedure as for U.S. banks, then, we generate 20,000 bootstrap samples — 10,000 before 2011 Q4, and 10,000 after — and then generate two sets of 10,000 estimates of the reversion rates $-\alpha_k$ from which we can calculate confidence intervals and p-values.

In Figure 13, we report point estimates and 95% confidence intervals for the relative growth rates α_k for different ranked European commercial banks. The figure shows that the average α_k for many middle-sized banks in each time period is outside of the other time period's 95% confidence interval. In the right panel of Figure 14, we report p-values for the hypothesis of no decrease (no increase) in European asset relative growth rates α_k (reversion rates $-\alpha_k$) after 2011 Q4. These p-values are computed in the same way as for the U.S. bank idiosyncratic asset volatility p-values shown in the left panel of Figure 14. According to the right panel of Figure 14, the higher asset reversion rates after 2011 Q4 for many middle-sized European banks is statistically significant at both the 1% and 5% levels. This result is notable given the small sample size of our European bank data set.

Rank-Based Methods versus Gibrat's Law

In order to examine the robustness of our estimated parameters σ_k and α_k, we perform a simple out-of-sample analysis using our estimates of these parameters for the U.S. banking sector as shown in the left panels of Figures

[11] Even though we assume that the bootstrap estimates of σ_k are independent across all ranks k, some rank-dependence is in fact introduced into these estimates when they are smoothed across ranks.

2 and 3.[12] The goal is to examine the out-of-sample accuracy of our rank-based methods as compared to Gibrat's law. For both the 1986 Q2 - 1998 Q3 and 1998 Q4 - 2016 Q3 time periods, we use the first half of the time period to estimate parameters σ_k and α_k using the procedure outlined in Section 2. We also use these half-samples to estimate these parameters while imposing Gibrat's law, so that $\sigma_1 = \sigma_2 = \cdots = \sigma_{N-1} > 0$ and $\alpha_1 = \alpha_2 = \cdots = \alpha_{N-1} < 0$. Thus, we have two sets of parameters σ_k and α_k to compare — the first is unconstrained and estimated using the procedure of Section 2, and the second is constrained and estimated while imposing Gibrat's law.

The next step is to generate estimates of bank asset shares $\theta_{(k)}$ using equation (2) from Theorem 2.1. We generate these predicted asset shares using both the unconstrained and constrained (Gibrat's law) sets of parameters σ_k and α_k estimated over the first half of both the 1986 Q2 - 1998 Q3 and 1998 Q4 - 2016 Q3 time periods. These predicted shares can then be compared, out of sample, to the observed asset shares over the second half of the 1986 Q2 - 1998 Q3 and 1998 Q4 - 2016 Q3 time periods. The root mean squared error (RMSE) between predicted and observed log asset shares using the unconstrained estimates of the parameters σ_k and α_k is 1.8931 over the second half of the 1986 Q2 - 1998 Q3 time period, and 0.2988 over the second half of the 1998 Q4 - 2016 Q3 time period. In contrast, the constrained estimates of these parameters that impose Gibrat's law generate RMSEs between predicted and observed log asset shares of 3.4267 and 7.3403 over, respectively, the second half of the 1986 Q2 - 1998 Q3 and 1998 Q4 - 2016 Q3 time periods. According to these results, then, the extra flexibility of our nonparametric, rank-based methods generates lower out-of-sample prediction errors than Gibrat's law. Furthermore, over the 1998 Q4 - 2016 Q3 time period, this goodness-of-fit improvement is quite substantial.

Figure 15 helps us to visualize the dramatically improved fit of our rank-based framework. This figure plots the predicted distribution of U.S. bank assets both with and without imposing Gibrat's law when estimating the parameters σ_k and α_k. The distribution curves shown in Figure 15 are different from the out-of-sample goodness-of-fit analysis described above, since

[12] We focus on the estimated parameters for the U.S. banking sector because of the larger sample period for this data set (1986-2016 rather than 2008-2016). If we perform a similar out-of-sample analysis using half-samples of our European bank asset data, then we must estimate the parameters σ_k and α_k using as few as eight quarters of data. This would not be an appropriate application of our rank-based empirical methods.

the parameters σ_k and α_k (both with and without imposing Gibrat's law) are estimated using the full 1998 Q4 - 2016 Q3 sample period. Nonetheless, similar to the out-of-sample analysis, this figure shows that Gibrat's law again fails to accurately describe the distribution of U.S. bank assets. In fact, the distribution implied by Gibrat's law in Figure 15 has the top 10 banks holding 98% of total assets, while the true distribution has this share at 62% on average from 1998 Q4 - 2016 Q3.

	Hill	Rank - 1/2
Gramm-Leach-Bliley Act Dummy	0.417***	0.490***
	(0.018)	(0.020)
Constant	1.130***	0.851***
	(0.014)	(0.015)
R-Squared	0.812	0.834
N	122	122

* p¡0.05, ** p¡0.01, *** p¡0.001

Table 1: Time-series regressions of Hill and Rank - 1/2 OLS estimates of the slope of the log-log plot of U.S. bank size versus rank.

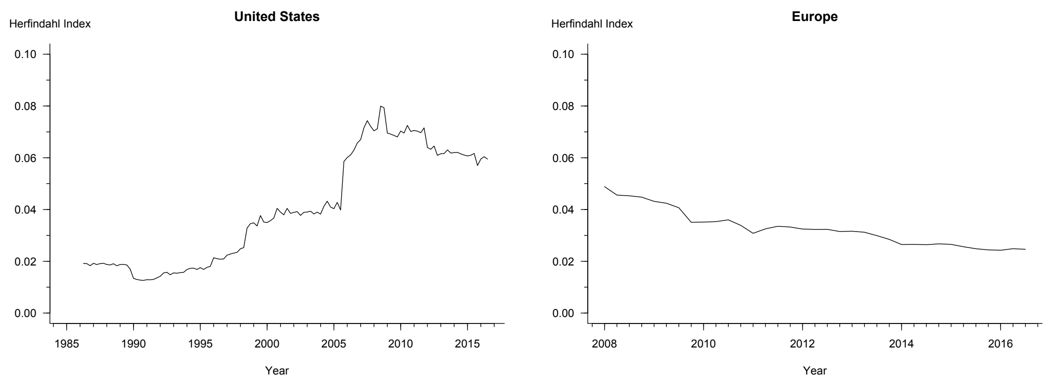

Figure 10: Herfindahl index for U.S. bank assets, 1986-2016 (left panel), and European bank assets, 2008-2016 (right panel).

Figure 11: Minus the estimated slope of the log-log plot of size versus rank for the top 100 U.S. banks, 1986-2016 (left panel), and the top 40 European banks, 2008-2016 (right panel).

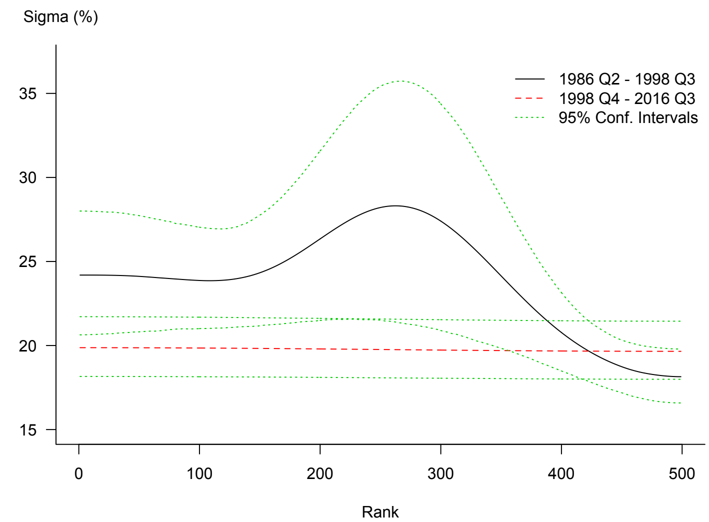

Figure 12: Standard deviations of idiosyncratic asset volatilities (σ_k) and 95% confidence intervals for different ranked U.S. banks.

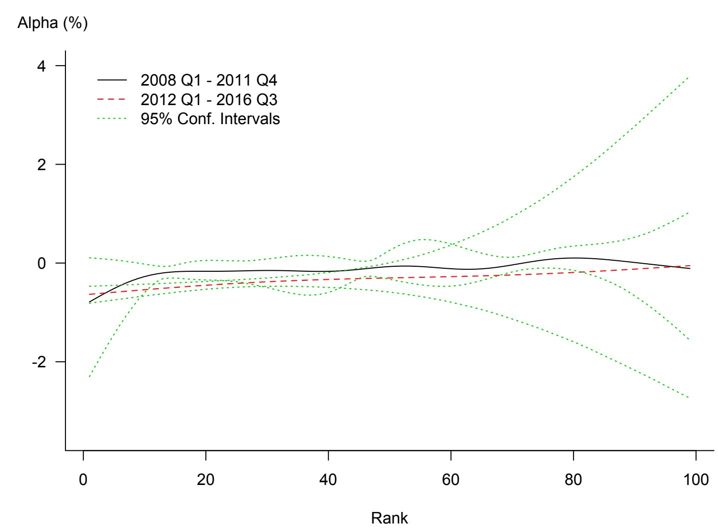

Figure 13: Minus the revision rates (α_k) and 95% confidence intervals for different ranked European banks.

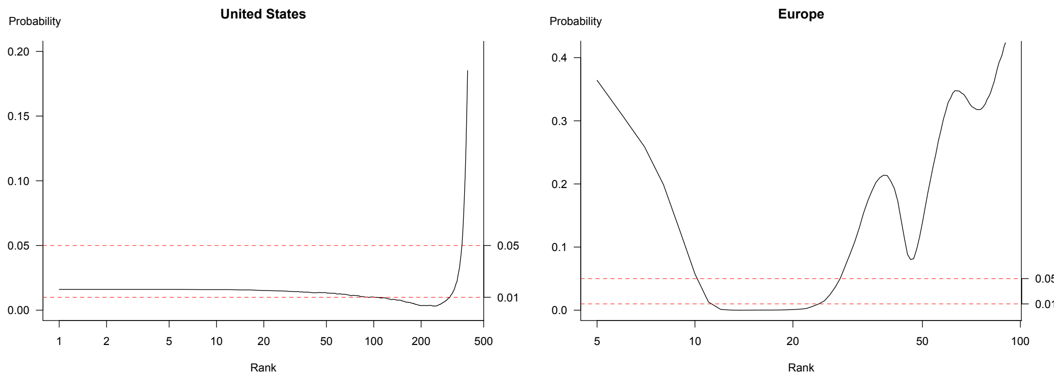

Figure 14: Probability that σ_k from 1986 Q2 - 1998 Q3 is less than or equal to σ_k from 1998 Q4 - 2016 Q3 for different ranked U.S. banks (left panel). Probability that α_k from 2008 Q1 - 2011 Q4 is less than or equal to α_k from 2012 Q1 - 2016 Q3 for different ranked European banks (right panel).

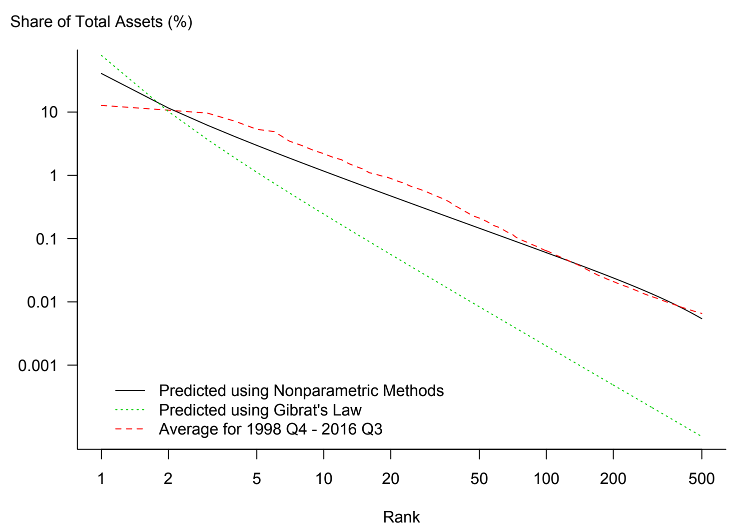

Figure 15: Shares of total assets held by the 500 largest U.S. banks, 1998 Q4 - 2016 Q3, as compared to the predicted shares using nonparametric dynamic power law methods and the predicted shares when imposing Gibrat's law.