Instability and Concentration in the Distribution of Wealth

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Income and Wealth Distributions

- Significant concentration of income and wealth observed worldwide
- Richest 1% holds 33% of wealth, earns 25% of income in U.S.
  - Atkinson, Piketty, and Saez (2011), Wolff (2012)
- Gini coefficients of 0.55 for income and 0.80 for wealth in U.S.
  - Díaz-Giménez, Quadrini, Ríos-Rull, and Rodríguez (2002)
  - Davies, Sandström, Shorrocks, and Wolff (2011)
- Many different explanations have been proposed
  - Krussel and Smith (1998), Quadrini (2000), De Nardi (2004)

Idiosyncratic Investment Risk

- Uninsurable idiosyncratic investment risk another possible explanation
  - Angeletos and Calvet (2006), Benhabib, Bisin, and Zhu (2011)
  - Generates realistic stationary Pareto distribution of wealth
- Strong empirical motivation for uninsurable investment risk
  - Housing: Case and Shiller (1989), Flavin and Yamashita (2002)
  - Private equity: Moskowitz and Vissing-Jorgensen (2002)
  - Together, these make up more than 50% of total U.S. household wealth: Bertaut and Starr-McCluer (2002) and Wolff (2012)

Key Elements of the Model

1. Heterogeneous households that face idiosyncratic investment risk
   - Can invest in individual-specific asset subject to uninsurable risk

2. No redistributive mechanisms
   - Broad interpretation: any factor that affects wealthy households and poor households differently
     - Government tax or fiscal policies
     - Limited intergenerational transfers

3. Forward-looking households that behave optimally
   - Choose how much to consume and how to invest savings
   - Efficient and rational model outcome

Instability and Concentration

- Equilibrium distribution of wealth is not stationary
  - Right-skewness increases over time
  - Wealth eventually concentrates entirely at the top
  - Instability characterized using recent results from mathematical finance
- Luck alone generates diverging levels of wealth
  - Households have same abilities, opportunities, and preferences
- Unlike previous literature, no redistributive mechanisms in this setup
  - Explicit, implicit redistributive mechanisms play crucial stabilizing role

Equilibrium Wealth Dynamics

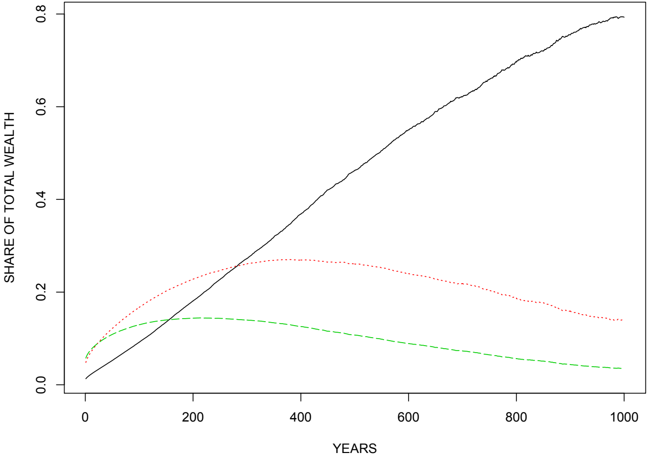

Figure: The shares of total wealth held by the wealthiest 1% (solid black line), the wealthiest 1-5% (dotted red line), and the wealthiest 5-10% (dashed green line).

Outline

Heterogeneous Households

- Economy is populated by N infinitely-lived households
  - At each $t \in [0, \infty)$, each household solves savings-consumption problem
  - Each household receives labor income equal to λ throughout its life
- Households have two investment options:
  - Risk-free asset that pays a return of r
  - Individual-specific asset subject to idiosyncratic risk
- For all $i = 1, \ldots, N$, price of individual-specific risky asset given by

$$dP_i(t) = \alpha P_i(t)\, dt + \sigma P_i(t)\, dB_i(t)$$

  - Uninsurable risk: Brownian motions B_i independent of each other

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Household Maximization Problems

Each household i has CRRA utility from consumption:

$$J(w,t) = \max_{c_i(t),\phi_i(t)} E_t\left[\int_t^\infty \frac{c_i^{1-\gamma}(s)}{1-\gamma} e^{-\rho s}\, ds\right]$$

$$\text{s.t.} \quad dw_i(s) = \left[rw_i(s) + (\alpha - r)\phi_i(s)w_i(s) - c_i(s) + \lambda\right] ds + \sigma\phi_i(s)w_i(s)\, dB_i(s)$$

$c_i(t)$: Consumption

$\phi_i(t)$: Fraction of wealth $w_i(t)$ invested in risky asset

$\gamma \geq 1$: Coefficient of relative risk aversion

$\rho > 0$: Discount rate

Consumption and Investment

Proposition

For all households $i = 1, \ldots, N$, the policy functions $c_i(t)$ and $\phi_i(t)$ are given by

$$c_i(t) = \left(\frac{\rho - (1-\gamma)r}{\gamma} - \frac{(1-\gamma)(\alpha - r)^2}{2\gamma^2\sigma^2} \right) \left(w_i(t) + \frac{\lambda}{r} \right),$$

$$\phi_i(t) = \frac{(\alpha - r)\left(w_i(t) + \frac{\lambda}{r} \right)}{w_i(t)\gamma\sigma^2}.$$

- Household maximization problems similar to Merton (1969)
  - Standard intuition behind optimal $c_i(t)$ and $\phi_i(t)$
  - $\frac{\lambda}{r}$ equal to households' discounted future labor income

Household Wealth Dynamics

Let $x_i(t)$ be total wealth of household $i = 1, \ldots, N$ at time t, so that

$$x_i(t) = w_i(t) + \frac{\lambda}{r}.$$

Given $c_i(t)$ and $\phi_i(t)$, then

$$dx_i(t) = \underbrace{\left(\frac{r-\rho}{\gamma} + \frac{(1+\gamma)(\alpha-r)^2}{2\gamma^2\sigma^2}\right)}_{\Lambda} x_i(t)\,dt + \underbrace{\left(\frac{\alpha-r}{\gamma\sigma}\right)}_{\Gamma} x_i(t)\,dB_i(t).$$

Substitution then yields

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Shares of Total Wealth

Let $\theta_i(t)$ be share of total wealth held by household i:

$$\theta_i(t) = \frac{x_i(t)}{x_1(t) + \cdots + x_N(t)}$$

Let $\theta_{\max}(t)$ be share of total wealth held by wealthiest household:

$$\theta_{\max}(t) = \max\left(\theta_1(t), \ldots, \theta_N(t)\right)$$

Note that

- Wealth shares add up to one: $\theta_1(t) + \cdots + \theta_N(t) = 1$
- $0 < \theta_i(t) < 1$ for all $i = 1, \ldots, N$, and so $\theta_{\max}(t) < 1$

Theorem (Wealth Dynamics Theorem)

If households face uninsurable idiosyncratic investment risk, then the share of the economy's total wealth held by the wealthiest single household, $\theta_{\max}$, satisfies

$$\lim_{T\to\infty} \frac{1}{T}\int_0^T \theta_{\max}(t)\,dt = 1, \quad \text{a.s.}$$

- Economy's wealth eventually concentrates at very top of distribution
  - Wealth distribution is not stationary
  - Limit of time-average of $\theta_{\max}(t)$, not of $\theta_{\max}(t)$ itself
- Similar result obtains in more general settings
  - Time-varying abilities and patience
  - As long as wealthy households do not have lower expected performance

Wealth Dynamics Theorem

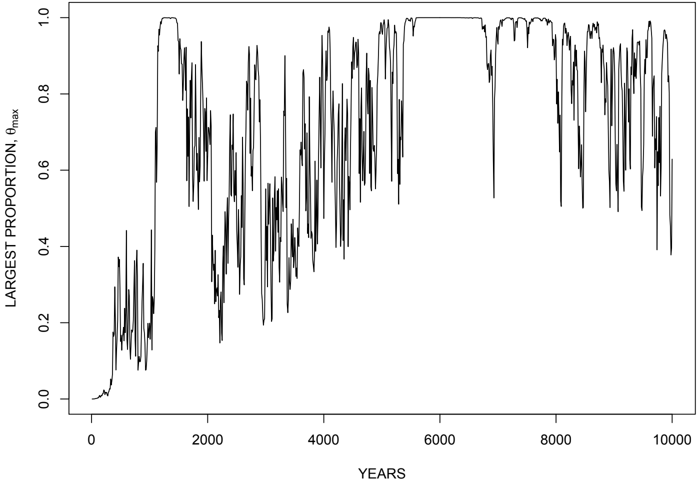

Figure: The share of total wealth held by the wealthiest household.

Equality of Opportunity and Ability

How surprising is this result?

All households in the economy are essentially identical:

- Same abilities and opportunities
  - Labor income, expected returns of individual-specific assets
- Same patience
  - Preferences for consumption over time

The implication is that luck alone, in the form of high *realized* investment returns, generates this extreme divergence.

No Redistributive Mechanisms

- No redistributive mechanisms, broadly defined, causes concentration
  - Explicit: government tax or fiscal policies
  - Implicit: limited intergenerational transfers
- Crucial distinction between this and all other setups
  - Champernowne (1953), Gabaix (2009), Benhabib, Bisin, & Zhu (2011)
- What is a redistributive mechanism, exactly?
  - Wealth dynamics for each household i:

$$dx_i(t) = \Lambda x_i(t)\,dt + \Gamma x_i(t)\,dB_i(t)$$

  - Redistributive mechanism means lower Λ for wealthier households

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Implications

- Redistributive mechanisms play crucial stabilizing role in economy
  - Many ways this can occur, but stability requires *something*
- What are the important real-world redistributive mechanisms?
- Why do wealthy households grow more slowly than poor households?
  - Taxes, fiscal policy, intergenerational transfers, psychology
  - A possible stabilizing role for redistributive income/estate taxes?

Parameterizing the Model

Wealth dynamics for each household i:

$$dx_i(t) = \Lambda x_i(t)\,dt + \Gamma x_i(t)\,dB_i(t)$$

Consequently, all that matters for simulations is

$$\Gamma = \frac{\alpha - r}{\gamma\sigma}$$

Benchmark parameterization:

- $N = 1,000,000$ households, $\alpha = 0.07, \sigma = 0.2$, and $r = 0.03, \gamma = 2$
  - Flavin & Yamashita (2002), Moskowitz & Vissing-Jorgensen (2002), Angeletos (2007), Benhabib, Bisin, & Zhu (2011)
- These values yield $\Gamma = 0.1$

Benchmark Parameterization: Wealth Shares

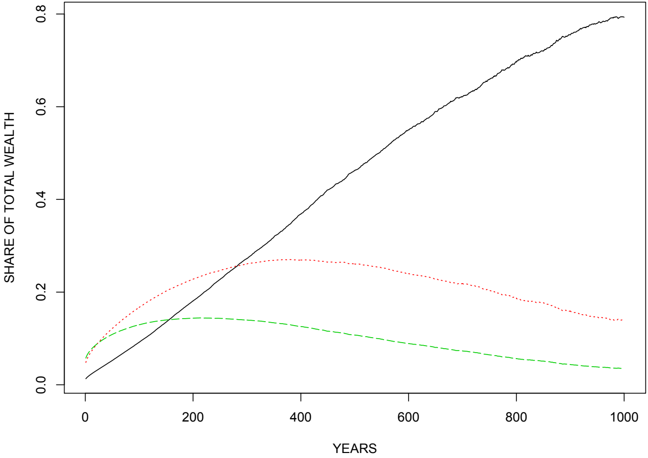


Figure: The shares of total wealth held by the wealthiest 1% (black), the wealthiest 1-5% (dotted red), and the wealthiest 5-10% (dashed green). ($\Gamma = 0.1$)

Benchmark Parameterization: Gini Coefficient

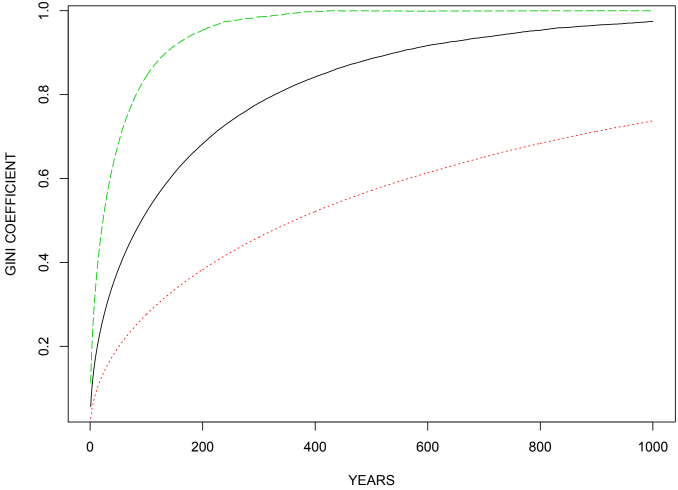


Figure: The Gini coefficient of the economy for $\Gamma = 0.1$.

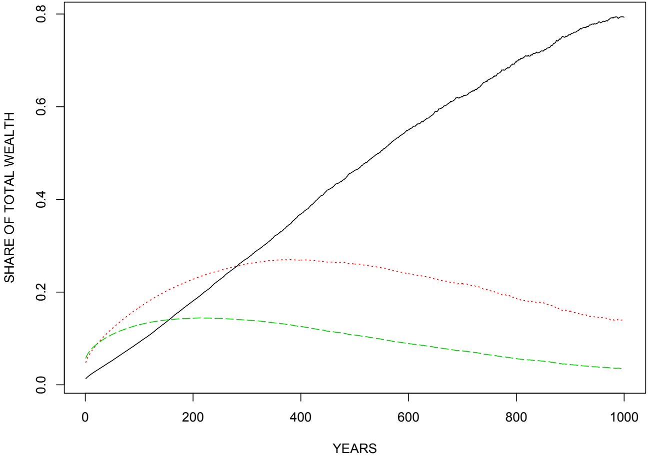
Key Role of Idiosyncratic Investment Risk

- Uninsurable investment risk and rate of wealth concentration linked
  - Increased exposure to idiosyncratic risk leads to faster wealth concentr.
- Recall, all that matters for simulations is

$$\Gamma = \frac{\alpha - r}{\gamma\sigma}$$

- Benchmark parameterization: $\Gamma = 0.1$
- Two alternative parameterizations
  - High exposure to idiosyncratic investment risk: $\Gamma = 0.2$
  - Low exposure to idiosyncratic investment risk: $\Gamma = 0.05$

Benchmark Parameterization



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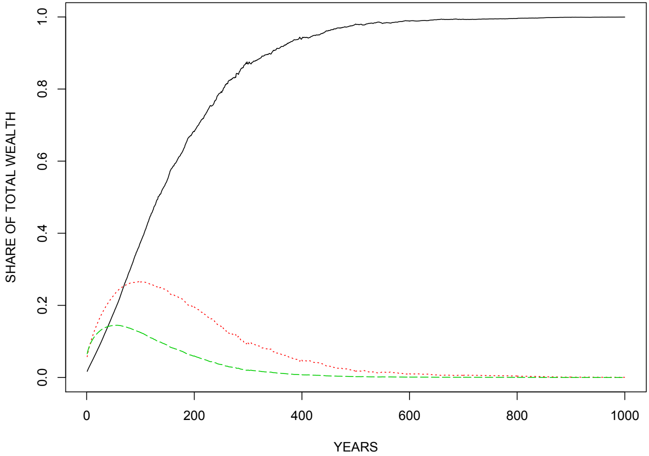
More Exposure to Investment Risk



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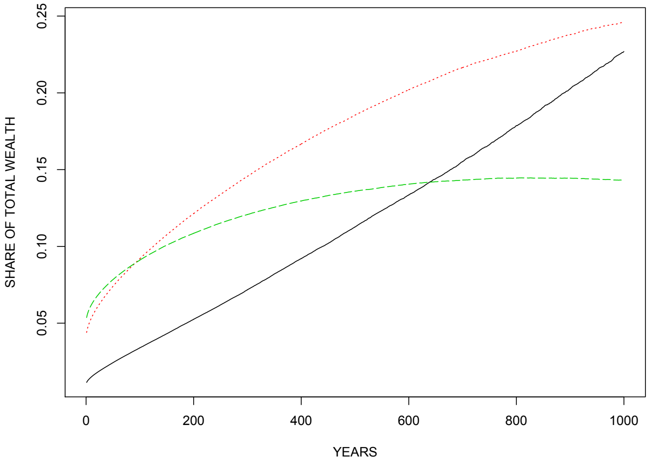
Less Exposure to Investment Risk



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Gini Coefficients and Exposure to Investment Risk

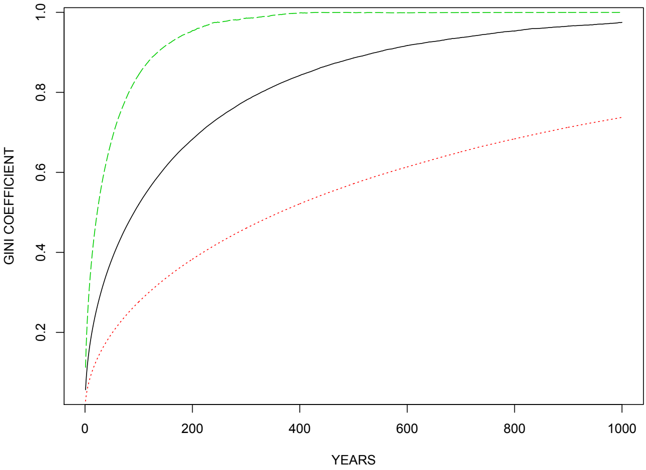

Figure: The Gini coefficient of the economy for $\Gamma = 0.1$ (solid black line), $\Gamma = 0.05$ (dotted red line), and $\Gamma = 0.2$ (dashed green line).

Outline

Summary

- Three key elements to the model:
  1. Heterogeneous households that face idiosyncratic investment risk
  2. No redistributive mechanisms
  3. Forward-looking households that behave optimally

- In this setting, equilibrium distribution of wealth is not stationary
  - Increasing right-skewness, eventually full concentration at the top

- Instability and concentration a consequence of luck alone
  - Different realized investment returns

Implications and Extensions

- In the presence of uninsurable investment risk, the natural tendency of wealth is to concentrate
  - Luck, in the form of different realized investment returns
- Redistributive mechanisms play crucial stabilizing role in economy
  - Which mechanisms are most important in modern economies?
  - Intergenerational transfers, taxes, something else?
- Analytic techniques used in paper easily extended to stable economies
  - Household-by-household description of equilibrium wealth distribution

The End

Thank You