The Distributional Effects of Progressive Capital Taxes

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Abstract

Rising income and wealth inequality since the 1980s has spurred much research examining the underlying causes and potential policy responses. Among the more controversial, Piketty (2014) proposes a progressive capital tax as a response to rising top wealth shares around the world. This paper introduces rank-based econometric methods for dynamic power laws as a tool for estimating the effect of progressive capital taxes on the distribution of wealth. We use available data to approximate the shaping econometric factors of the U.S. wealth distribution, and then consider the effects of progressive capital taxes on these shaping factors. This yields empirical estimates of the distributional effects of progressive capital taxes under different assumptions about the impact of these taxes on household behavior. In most scenarios, we find that a small tax levied on just 1% of households would substantially reshape the U.S. wealth distribution and reduce inequality.

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1 Introduction

Recent trends in income and wealth inequality have drawn much attention from both academic researchers and the general public. The detailed empirical work of Atkinson et al. (2011), Davies et al. (2011), and Saez and Zucman (2016), among others, documents these trends for many different countries around the world and has prompted a substantive debate about their underlying causes and the appropriate policy responses, if any. Piketty (2014), for example, generated much controversy after proposing a simple mechanism based on the difference between returns to capital and income growth to explain the long-run tendency of inequality to rise around the world (Acemoglu and Robinson, 2014; Krussel and Smith, 2015). He then went on to suggest a progressive capital tax for Europe as a policy response.

The debate surrounding progressive capital taxes has so far focused on the potentially significant economic distortions that could result from such a tax versus the substantial revenue that it would likely generate (Piketty, 2014). A quantitative assessment of the potential for progressive capital taxes to reduce inequality, however, has been notably absent from the discussion. This paper addresses this omission. We accomplish this using rank-based empirical methods for dynamic power law distributions first introduced to economics by Fernholz (2016a). These methods allow us to consider the distributional effects of a progressive capital tax similar to that proposed by Piketty (2014) without making difficult assumptions about the underlying causes or consequences of rising inequality. We consider several different assumptions about the current trajectory of the U.S. wealth distribution as well as the impact of capital taxes on household behavior, but in almost all of these scenarios we find that a progressive capital tax of 1-2% levied on just 1% of households substantially reshapes the U.S. distribution of wealth and reduces inequality

We consider a general random growth economy consistent with incomplete markets models of inequality (Aiyagari, 1994; Benhabib et al., 2011; Fernholz, 2016b). In contrast to this literature, however, we impose no parametric structure on the underlying processes of household wealth accumulation and do not model or estimate these processes directly. Despite the minimal structure of our approach, we use new results to obtain a closed-form rank-by-rank characterization of the stationary distribution of wealth. These results yield an asymptotic identity describing the stationary distribution in terms of two econometric factors—the reversion rates and idiosyncratic volatilities of wealth for different ranked households (Fernholz, 2016a). The first factor measures the intensity of cross-sectional mean reversion of household wealth. The second factor has been analyzed as a determinant of wealth inequality in a number of instances (Benhabib et al., 2015; Gabaix et al., 2016).

In order to estimate the distributional effects of progressive capital taxes, two major challenges must be overcome. First, it is necessary to understand how such taxes impact household saving and portfolio choice as well as household efforts to evade the taxes by, for example, hiding wealth offshore (Zucman, 2013). Second, it is necessary to analyze how altering the growth rates of wealth at different ranks in the distribution affects the stationary distribution of wealth. This paper provides methods that address this second challenge. We also consider three simple scenarios that encompass a range of potential household responses to capital taxes.

One of the advantages of our nonparametric framework is that it can accurately replicate any empirical distribution. Using the detailed new wealth shares data of Saez and Zucman (2016), we construct such a match for the 2012 U.S. wealth distribution. This is accomplished by first using estimates about idiosyncratic capital income risk from Moskowitz and Vissing-Jorgensen (2002) to approximate the idiosyncratic volatility of household wealth, and then using these estimates to infer the implied values for the reversion rates of wealth for different ranked households. These rank-based reversion rates generate a precise match of a stationary 2012 U.S. wealth distribution.

We use these 2012 estimates of reversion rates and idiosyncratic volatilities to analyze the distributional effects of a progressive capital tax. Following Piketty (2014), we consider a progressive capital tax of 1-2% levied on 1% of households in the economy, and show that in all scenarios we consider this capital tax substantially reduces inequality and reshapes the distribution of wealth. For example, if the 2012 U.S. wealth distribution is assumed to be stationary, then our estimates suggest that this tax would reduce inequality to levels comparable to those observed in the U.S. in the 1970s. The reason for this large distributional effect is that the top 1% of households that pay the tax hold between 40-85% of total wealth in all scenarios we consider. As a consequence, a small capital tax imposed on just 1% of households will in fact affect a large fraction of the economy's total wealth. Finally, we stress that this result is not a statement about total welfare and not an endorsement of a progressive capital tax. Instead, our analysis of the distributional effects of progressive capital taxes is meant only to enhance our overall understanding of the implications of such a policy.

The rest of this paper is organized as follows. Section 2 presents an overview of our rank-based empirical framework, and Section 3 reports estimates of rank-based reversion rates and idiosyncratic volatilities that generate an exact match of the 2012 U.S. wealth distribution. Section 4 presents estimates of the effect of progressive capital taxes on inequality. Section 5 concludes.

2 A Rank-Based Approach to Inequality

We use the nonparametric empirical methods for dynamic power law distributions detailed by Fernholz (2016a) to characterize the distribution of wealth.[1]

2.1 Household Wealth Dynamics

Consider an economy that is populated by $N > 1$ households. Time is continuous and denoted by $t \in [0, \infty)$, and uncertainty in this economy is represented by a filtered probability space $(\Omega, \mathcal{F}, \mathcal{F}_t, P)$. Let $\mathbf{B}(t) = (B_1(t), \ldots, B_M(t))$, $t \in [0, \infty)$, be an M-dimensional Brownian motion defined on the probability space, with $M \geq N$. We assume that all stochastic processes are adapted to $\{\mathcal{F}_t; t \in [0, \infty)\}$, the augmented filtration generated by $\mathbf{B}$.

The total wealth of each household $i = 1, \ldots, N$ in this economy is given by the process w_i. Each of these wealth processes evolves according to the stochastic differential equation

$$d \log w_i(t) = \mu_i(t)\, dt + \sum_{z=1}^{M} \delta_{iz}(t)\, dB_z(t), \tag{2.1}$$

where μ_i and δ_{iz}, $z = 1, \ldots, M$, are measurable and adapted processes. The growth rates and volatilities μ_i and δ_{iz} are general and essentially unrestricted, having only to satisfy a few basic regularity conditions.[2] Equation (2.1) together with these regularity conditions implies that the household wealth processes are general Itô processes, which represent a

[1]For brevity, we refer directly to Fernholz (2016a) on several occasions in this section and leave out numerous details and proofs that are available there.

[2]These conditions ensure basic integrability of equation (2.1) and require that no two households' wealth holdings are fully correlated over time. See Appendix A of Fernholz (2016a) for details.

broad class of stochastic processes. According to the martingale representation theorem (Nielsen, 1999), many different continuous processes for household wealth can be written in the nonparametric form of equation (2.1).

The general, nonparametric structure of our approach implies that almost all previous empirical and theoretical models of income and wealth represent special cases of equation (2.1). Indeed, most of the theoretical literature on wealth distribution assumes that households are ex-ante symmetric and hence that the growth rate parameters μ_i and the standard deviation parameters δ_{iz} in equation (2.1) do not persistently differ across households (Benhabib et al., 2011; Fernholz, 2016b). Even when the parameters μ_i and δ_{iz} do persistently differ across households, such as in much of the empirical literature on income processes (Guvenen, 2009), this heterogeneity is usually constrained by some specific parametric structure. In this sense, then, our framework encompasses and extends the previous literature.

One of the necessary regularity assumptions ensures that no two households' wealth dynamics are fully correlated over time. In other words, markets are incomplete and all households face at least some idiosyncratic risk to their wealth holdings. This assumption is consistent with both the Bewley models of uninsurable labor income risk (Aiyagari, 1994; Krussel and Smith, 1998) and the more recent literature that considers uninsurable capital income risk (Angeletos, 2007; Benhabib et al., 2011). This section's results characterize the effect of idiosyncratic risk to households' wealth holdings on inequality.

In order to characterize the distribution of wealth in this economy, it is necessary to consider the dynamics of household wealth by rank. To accomplish this, we introduce notation for household rank based on wealth holdings. For $k = 1, \ldots, N$, let $w_{(k)}(t)$ represent the wealth holdings of the k-th wealthiest household in the economy at time t, so that

$$\max(w_1(t), \ldots, w_N(t)) = w_{(1)}(t) \geq w_{(2)}(t) \geq \cdots \geq w_{(N)}(t) = \min(w_1(t), \ldots, w_N(t)). \quad (2.2)$$

Next, let $\theta_{(k)}(t)$ be the share of total wealth held by the k-th wealthiest household at time t, so that for $k = 1, \ldots, N$,

$$\theta_{(k)}(t) = \frac{w_{(k)}(t)}{w(t)}, \quad (2.3)$$

where $w(t) = w_1(t) + \cdots + w_N(t)$ denotes the total wealth of all households in the economy.

To be able to link household rank (denoted by k) to household index (denoted by i), let

p_t be the random permutation of $\{1, \ldots, N\}$ such that for $1 \leq i, k \leq N$,

$$p_t(k) = i \quad \text{if} \quad w_{(k)}(t) = w_i(t). \tag{2.4}$$

This definition implies that $p_t(k) = i$ whenever household i is the k-th wealthiest household in the economy. In order to describe the dynamics of the ranked wealth processes $w_{(k)}$ and ranked wealth share processes $\theta_{(k)}$, $k = 1, \ldots, N$, it is necessary to introduce the notion of a local time. Both Karatzas and Shreve (1991) and Fernholz (2016a) detail how the local time process Λ_x measures the amount of time the process x spends near zero.

According to Fernholz (2016a), for all $k = 1, \ldots, N$, the dynamics of the ranked wealth processes $w_{(k)}$ and ranked wealth share processes $\theta_{(k)}$ are given by[3]

$$d \log w_{(k)}(t) = d \log w_{p_t(k)}(t) + \frac{1}{2} d\Lambda_{\log w_{(k)} - \log w_{(k+1)}}(t) - \frac{1}{2} d\Lambda_{\log w_{(k-1)} - \log w_{(k)}}(t), \tag{2.5}$$

and

$$d \log \theta_{(k)}(t) = d \log \theta_{p_t(k)}(t) + \frac{1}{2} d\Lambda_{\log \theta_{(k)} - \log \theta_{(k+1)}}(t) - \frac{1}{2} d\Lambda_{\log \theta_{(k-1)} - \log \theta_{(k)}}(t). \tag{2.6}$$

To understand equation (2.5), note that the local time terms in this equation only contribute to $w_{(k)}(t)$ if the k-th wealthiest household's wealth either falls to the level of the $(k+1)$-th wealthiest household's wealth (this corresponds to $\Lambda_{\log w_{(k)} - \log w_{(k+1)}}$) or rises to the level of the $(k-1)$-th wealthiest household's wealth (this corresponds to $\Lambda_{\log w_{(k-1)} - \log w_{(k)}}$). In the former case, the positive local time term ensures that the k-th wealthiest household is always wealthier than the $(k+1)$-th wealthiest household. Conversely, in the latter case, the negative local time term ensures that the k-th wealthiest household is always less wealthy than the $(k-1)$-th wealthiest household. A similar logic applies to equation (2.6) for the ranked wealth share processes $\theta_{(k)}$.

2.2 Stationary Distribution of Wealth

Let α_k equal the time-averaged limit of the expected growth rate of wealth for the k-th wealthiest household at time t, $\mu_{p_t(k)}$, relative to the expected growth rate of wealth for the

[3]Throughout this paper, we shall use the convention that $\Lambda_{\log w_{(0)} - \log w_{(1)}}(t) = \Lambda_{\log w_{(N)} - \log w_{(N+1)}}(t) = 0$. We shall also write $dx_{p_t(k)}(t)$ to refer to the process $\sum_{i=1}^{N} 1_{\{i = p_t(k)\}} dx_i(t)$.

whole economy, which we denote by μ, so that

$$\alpha_k = \lim_{T\to\infty} \frac{1}{T} \int_0^T \left(\mu_{p_t(k)}(t) - \mu(t)\right) dt, \tag{2.7}$$

for $k = 1, \ldots, N$. We shall refer to $-\alpha_k$ as reversion rates, which are a rough measure of the rate at which wealth cross-sectionally reverts to the mean. These parameters indirectly incorporate all relevant aspects of the economic environment.

In a similar manner, we wish to define the time-averaged limit of the volatility of the process $\log\theta_{(k)} - \log\theta_{(k+1)}$, which measures the relative wealth holdings of adjacent households in the distribution of wealth. For all $k = 1, \ldots, N-1$, let σ_k be given by

$$\sigma_k^2 = \lim_{T\to\infty} \frac{1}{T} \int_0^T \sum_{z=1}^{M} \left(\delta_{p_t(k)z}(t) - \delta_{p_t(k+1)z}(t)\right)^2 dt. \tag{2.8}$$

The relative growth rates α_k together with the volatilities σ_k entirely determine the shape of the stationary distribution of wealth in this economy. Finally, for all $k = 1, \ldots, N$, let

$$\kappa_k = \lim_{T\to\infty} \frac{1}{T} \Lambda_{\log\theta_{(k)} - \log\theta_{(k+1)}}(T). \tag{2.9}$$

The *stable version* of the process $\log\theta_{(k)} - \log\theta_{(k+1)}$ is the process $\log\theta^*_{(k)} - \log\theta^*_{(k+1)}$ defined by

$$d\left(\log\theta^*_{(k)}(t) - \log\theta^*_{(k+1)}(t)\right) = -\kappa_k\, dt + d\Lambda_{\log\theta^*_{(k)} - \log\theta^*_{(k+1)}}(t) + \sigma_k\, dB(t), \tag{2.10}$$

for all $k = 1, \ldots, N-1$.[4] As long as the relative growth rates, volatilities, and local times that we take limits of in equations (2.7)-(2.9) do not change drastically and frequently over time, then the distribution of the stable versions of $\theta_{(k)}$ will accurately reflect the distribution of the true versions of these ranked wealth share processes. Throughout this paper, we shall assume that these limits do in fact exist.[5]

Theorem 2.1. *There is a stationary distribution of wealth in this economy if and only if* $\alpha_1 + \cdots + \alpha_k < 0$*, for* $k = 1, \ldots, N-1$*. Furthermore, if there is a stationary distribution of*

[4] For each $k = 1, \ldots, N-1$, equation (2.10) implicitly defines another Brownian motion $B(t)$, $t \in [0, \infty)$. These Brownian motions can covary in any way across different k.

[5] Note that the existence of the limits in equations (2.7)-(2.8) is a weaker assumption than the existence of a stationary distribution of wealth (Banner et al., 2005).

wealth, then for $k = 1, \ldots, N-1$, this distribution satisfies

$$E\left[\log \theta^*_{(k)}(t) - \log \theta^*_{(k+1)}(t)\right] = \frac{\sigma_k^2}{-4(\alpha_1 + \cdots + \alpha_k)}, \quad \text{a.s.} \tag{2.11}$$

Theorem 2.1 provides an analytic rank-by-rank characterization of the entire distribution of wealth. This is achieved despite minimal assumptions on the processes that describe the dynamics of household wealth over time. The empirical accuracy of this theorem has now been demonstrated in a number of different applications ranging from the distribution of market capitalization of U.S. stocks (Fernholz, 2002) to the distribution of assets of U.S. banks (Fernholz and Koch, 2016) to the distribution of relative commodity prices (Fernholz, 2016a).

Theorem 2.1 yields two important insights. First, it shows that an understanding of rank-based household wealth dynamics is sufficient to describe the entire distribution of wealth. It is not necessary to directly model and estimate household wealth dynamics by name, denoted by index i. Second, the theorem shows that the only two factors that affect the distribution of wealth are the rank-based reversion rates, $-\alpha_k$, and the rank-based idiosyncratic volatilities, σ_k. To understand the effect of a change in policy on inequality, then, it is necessary only to understand the effect that policy has on these reversion rates and volatilities. If quantitative estimates of these effects can be obtained, then Theorem 2.1 provides a quantitative description of the impact on inequality. This observation underlies our analysis of the distributional effects of progressive capital taxes in Section 4.

3 The U.S. Wealth Distribution

We seek estimates of the reversion rates and idiosyncratic volatilities that shape the stationary distribution of wealth according to equation (2.11) from Theorem 2.1. In order to generate these estimates, we use the detailed U.S. wealth distribution data of Saez and Zucman (2016).[6] Throughout this paper, we set the number of households in the economy N

[6] We use the wealth shares data of Saez and Zucman (2016) because of its great detail, especially for top shares. It should be noted, however, that the procedure of estimating the reversion rates and idiosyncratic

equal to one million. This number balances the need for realism with the need to perform computations and simulations in a reasonable amount of time.

Equation (2.11) establishes a simple relationship between inequality as measured by the expected difference $\log\theta^*_{(k)}(t) - \log\theta^*_{(k+1)}(t)$, the rank-based reversion rates, $-\alpha_k$, and the rank-based volatilities, σ_k. Ideally, we would use detailed panel data on individual households' wealth holdings over time to estimate the quantities α_k and σ_k, and then confirm that these estimates replicate the observed wealth shares $\theta_{(k)}$.[7] Unfortunately, a comprehensive panel data set on household wealth holdings in the U.S. does not yet exist. Given these data limitations, we instead choose to use estimates of the wealth shares $\theta_{(k)}$ and the rank-based volatilities σ_k to infer the values of the rank-based reversion rates $-\alpha_k$ via equation (2.11).

The first step in this process is to generate estimates of the rank-based volatilities σ_k. According to equation (2.8), these volatilities correspond to the time-averaged limit of the variance of the process $\log\theta_{(k)} - \log\theta_{(k+1)}$, which measures the relative wealth holdings of households that are adjacent in the wealth distribution. Although Fernholz (2016c) approximates these volatility parameters using previous research on the volatility of labor income and of the idiosyncratic component of capital income, for simplicity, in this paper we follow Gabaix et al. (2016) and focus entirely on idiosyncratic capital income risk.

The dynamic relationship between household wealth, capital income, labor income, and consumption can be written as

$$dw_i(t) = w_i(t)r_i(t)\,dt + \left(y_i(t) - c_i(t)\right)\,dt, \tag{3.1}$$

where y_i, c_i, and r_i denote, respectively, the after-tax labor income, consumption, and after-tax idiosyncratic return processes for household $i = 1, \ldots, N$. Equation (3.1) is a more general form of the stochastic differential equation used by Gabaix et al. (2016) to model household wealth dynamics over time. These authors show that with a simple proportional-consumption rule, the idiosyncratic volatility of the process $\log\theta_{(k)}$ is equal to the volatility of idiosyncratic after-tax capital income. Following Moskowitz and Vissing-Jorgensen (2002) and others, we set the volatility of after-tax capital income equal to 0.2. In terms of our rank-based, nonparametric framework, this implies that the squared volatility parameters

volatilities described in this section can be applied to any data set.

[7]This is the approach adopted by Fernholz (2002), Fernholz and Koch (2016), and Fernholz (2016a), for various different data sets.

σ_k^2 are equal to two times 0.2. This yields a value of 0.28 for the parameters σ_k, for all $k = 1, \ldots, N$.

Our choice to simplify and focus entirely on idiosyncratic capital income risk when approximating the volatility parameters σ_k follows Gabaix et al. (2016) and is motivated by two observations. First, in a Bewley incomplete markets model that also includes idiosyncratic capital income risk, Benhabib et al. (2015) show that capital income risk is the key long-run determinant of the equilibrium wealth distribution. This result is consistent with Gabaix et al. (2016), and provides both a theoretical and empirical justification for limiting our focus to undiversified household capital income risk. Second, Fernholz (2016c) approximates the parameters σ_k using the volatility of both labor income savings and capital income, yet still reports similar results for the distributional effects of a progressive capital tax. This suggests that labor income savings volatility is not a key economic determinant of the long-run wealth distribution. Over time, as new and better data on household wealth holdings become available, our understanding of the volatility parameters σ_k will improve.

The last step in estimating parameters that match the U.S. wealth distribution is to infer values for the reversion rates $-\alpha_k$ using equation (2.11). Normally, this is straightforward since the system of $N-1$ equations (2.11) together with the fact that $\alpha_1 + \cdots + \alpha_N = 0$ yields a solution. The problem, in this case, is that there are no wealth shares data that report the wealth holdings of each individual household in the economy $\theta_{(k)}$. Indeed, the data of Saez and Zucman (2016) report the wealth holdings of just a few subsets of U.S. households.

To fill in the missing wealth shares data, we assume a Pareto-like distribution of wealth in which the parameter of the Pareto distribution varies across different subsets of households in a way that matches the data. In fact, we find that varying the Pareto parameter across just three subsets of households achieves a nearly exact match of the 2012 U.S. wealth distribution as reported by Saez and Zucman (2016). Changing the Pareto parameter in this way is equivalent to assuming that the log-log plot of household rank versus household wealth holdings consists of three connected straight lines with different slopes.[8] A plot of this kind that achieves the closest possible match for the 2012 U.S. wealth distribution is

[8] More specifically, there is one straight line for the top 0.01% of households, that line connects to another straight line with a different slope for the top 0.01-10% of households, and that line connects to a third straight line with a different slope for the bottom 90% of households. Such a distribution generates a total absolute error relative to the true U.S. distribution of wealth in 2012 of just over 0.5%.

shown in Figure 1. This plot shows the value of log wealth shares $\theta_{(k)}$ versus the log of rank k. Once the household wealth shares $\theta_{(k)}$ are set, the rank-based reversion rates $-\alpha_k$ are inferred by solving the system of $N-1$ equations (2.11).

In the case of a standard Pareto distribution, a log-log plot as in Figure 1 appears as a single straight line with slope equal to the inverse of the Pareto parameter. Our approach is slightly more general and is preferred to restricting the wealth distribution to Pareto or lognormal since it allows for a more accurate match of the empirical distribution of wealth. This increased precision and flexibility highlights one of the advantages of the approach adopted here.

The process of estimating the model as described so far can be applied to any empirical distribution of wealth. This process yields implied values for the rank-based reversion rates $-\alpha_k$ using wealth shares data and estimates of the volatilities σ_k. Thus, if we use the 2012 U.S. wealth shares data of Saez and Zucman (2016) together with our estimate of σ_k, this generates implied values for the rank-based reversion rates.[9] These reversion rates generate a precise match of a stationary 2012 U.S. wealth distribution.

As the wealth shares data of Saez and Zucman (2016) show, however, stationarity of the 2012 U.S. distribution of wealth is unlikely. Indeed, a stationary distribution is one in which wealth shares are not trending up or down over time, but these data show that the share of total U.S. wealth held by the top 0.01% and 0.01-0.1% of households has been steadily rising since the mid-1980s. This issue is confronted by Gabaix et al. (2016), who in addition to analyzing the pace of rising U.S. income inequality, also provide estimates about the changing growth rates of wealth for the wealthiest U.S. households using data on the difference between the after-tax rate of return on wealth and the growth rate of income, $r-g$ (Piketty, 2014). According to Gabaix et al. (2016), the growth rates of wealth for the wealthiest U.S. households have increased by approximately 2% since 1980.

In order to address the non-stationarity of the 2012 U.S. wealth distribution, then, we follow Gabaix et al. (2016) and consider a second set of parameters α_k and σ_k in which the rank-based relative growth rates α_k for the wealthiest 1% of households are increased by 2% relative to their values in 1980. More precisely, we repeat the procedure of estimating the parameters α_k described above, but we do so using wealth shares data from 1980 rather than 2012. Then, we adjust the estimated α_k parameters for 1980 by increasing each α_k in

[9] Because this procedure produces one million different α_k values, we cannot directly report these estimates in the paper.

the top 1% by 2%. This yields a set of estimated parameters α_k and σ_k consistent with a non-stationary, transitioning 2012 U.S. wealth distribution.

The second columns of Tables 1 and 2 report the shares of wealth held by different ranked subsets of households for both the stationary and non-stationary parameterizations of the 2012 U.S. wealth distribution. In the case of the non-stationary parameterization, these shares correspond to the implied future stationary distribution of wealth according to equation (2.11) from Theorem 2.1. These non-stationary shares provide an implicit forecast of the future stationary U.S. wealth distribution. The stationary parameterization assumes that the 2012 distribution is stationary, and so the reported wealth shares in the second column of Table 1 are simply equal to those for the 2012 U.S. wealth distribution.

Figure 2 plots the distributions of wealth for the stationary and non-stationary parameterizations reported in the second column of Tables 1 and 2. Both the tables and the figure demonstrate that even small changes in the growth rate of wealth for the wealthiest U.S. households imply a substantial increase in wealth concentration over time. After all, even a small 2% increase in the growth rate of wealth for the wealthiest U.S. households in 1980—a time of much less inequality than 2012—causes top wealth shares to eventually rise higher than ever before in recorded history, as the second column of Table 2 shows. If this implicit forecast based on data from Gabaix et al. (2016) is correct, then U.S. wealth inequality should continue rising in the coming years.

4 Estimating the Distributional Effects of Progressive Capital Taxes

In the period since Piketty (2014) proposed a progressive capital tax in response to increasing income and wealth inequality, much of the debate surrounding this policy has centered on how it is likely to increase government revenues or distort economic outcomes rather than how it might affect the distribution of wealth. One of the contributions of this paper is to address this latter issue and provide estimates of the distributional effects of progressive capital taxes on the U.S. economy.

We analyze a simple progressive capital tax for the U.S. similar to the policy proposed by Piketty (2014). Our version of this tax sets the capital tax rate for the top 0.5% of households in the economy equal to 2% and the rate for the top 0.5-1% of households equal to 1%, while the remaining 99% of households are assumed to neither pay nor receive any tax or subsidy. As a consequence, none of the revenue generated from this progressive capital tax is redistributed to less wealthy households.[10] Based on the U.S. wealth distribution data for 2012, this progressive capital tax corresponds to a 2% rate for those households with total wealth greater than roughly $6 million and a 1% rate for those households with total wealth between $4 and $6 million.[11]

There are two key challenges to estimating the effects of this progressive capital tax on inequality. The first is to understand how it impacts household saving behavior and household efforts to evade the tax. This is a major challenge, as evidenced by the fact that almost no progress has been made towards solving portfolio optimization problems in which returns vary across different ranks in the wealth distribution. The second challenge is to understand how altering the growth rates of wealth at different ranks in the distribution affects the stationary wealth distribution. The central contribution of this paper is to provide an empirical framework that can address this second challenge. As more progress is made towards addressing the first challenge, these new insights can be combined with the empirical methods in this paper to produce superior estimates of the effects of general progressive capital taxes on inequality.

According to our results in Theorem 2.1, all that is necessary to estimate the distributional effects of the 1-2% capital tax based on Piketty (2014) are estimates of the effects of this tax on the rank-based reversion rates $-\alpha_k$ and volatilities σ_k. Because capital taxes are by design likely to have predictable effects on the growth rates of wealth for different wealth-ranked households, the empirical approach of Section 2 is uniquely suited to this task.

We consider three different scenarios for the impact of this 1-2% capital tax on wealth-ranked households' growth rates of wealth. Scenario 1 is a simple baseline scenario in which a capital tax rate of 1% on some subset of households in the economy reduces the growth

[10]It is straightforward to consider the distributional effects of such redistribution. However, since we find that these effects are quite small in our model of the U.S wealth distribution, we focus solely on the simple case of a tax without redistribution in this section.

[11]The basic progressive capital tax proposed by Piketty (2014) for Europe involves a 2% rate for those households with total wealth greater than €5 million, a 1% rate for those households with total wealth between €1 and €5 million, and no tax for the remaining households.

rate of wealth for those households by 1%. This baseline is a natural scenario to consider since the reduced savings of households in response to a capital tax, which magnifies the effect of the tax, are assumed in this scenario to be exactly balanced by households' ability to evade the tax, which diminishes the effect of the tax (Zucman, 2013).

The second scenario we consider assumes that a capital tax rate of 1% on some subset of households in the economy reduces the growth rate of wealth for those households by 0.5%. As a consequence, Scenario 2 assumes that households' evasion efforts have more impact than do households' reduced savings. Scenario 3 considers the opposite outcome. In particular, in this third scenario we assume that a capital tax rate of 1% on some subset of households reduces the growth rate of wealth for those households by 1.5%. This represents a situation in which households' reduced savings in response to the capital tax have more impact than do households' evasion efforts.

In terms of the parameters that shape the U.S. wealth distribution, Scenario 1 assumes that the 1-2% progressive capital tax that we consider reduces the rank-based relative growth rates α_k of those households that are taxed (the top 1%) by the rate at which they are taxed. Similarly, Scenarios 2 and 3 assume that this capital tax reduces the rank-based relative growth rates α_k of those households that are taxed by 0.5 and 1.5 times the rate at which they are taxed, respectively. Scenarios 1-3 cover a reasonable range of household responses to the tax, but they are by no means exhaustive. We stress, however, that alternative scenarios for the effects of this capital tax on household behavior and general equilibrium are easily evaluated using our empirical approach. All that needs to be done is to adjust the tax's impact on the growth rates of household wealth accordingly.

The distributional effects of a 1-2% progressive capital tax similar to that proposed by Piketty (2014) under Scenarios 1-3 are reported in Tables 1-2. These tables report these effects for both the stationary and non-stationary estimates of the parameters α_k discussed in Section 3. The results are also shown graphically in Figures 3-4. For both parameterizations and all three scenarios, the tables and figures show that a simple progressive capital tax imposed on just 1% of households in the economy substantially reshapes the U.S. distribution of wealth and reduces inequality. In the case of Scenarios 1 and 3, for example, the after-tax distribution of wealth for both the stationary and non-stationary parameterizations is similar to the distribution observed in the U.S. in the 1970s according to the historical wealth shares data of Saez and Zucman (2016). The fact that this period is one of the most egalitarian in the U.S. in the last century highlights just how significant this reduction in inequality is. Even for Scenario 2, in which the impact of the capital tax is greatly diminished by assumption, perhaps because of successful evasion efforts, the capital tax still reduces inequality substantially.

How is it that a 1-2% progressive capital tax levied on only 1% of households can reduce inequality so substantially? One might expect that such a large reduction in inequality requires that a larger subset of households be taxed. However, because the top 1% of households that pay the tax hold between 40-85% of total wealth depending upon the parameterization, a large fraction of the economy's total wealth is in fact affected by this progressive capital tax. As our results demonstrate, this large fraction of total wealth is sufficient for the tax to significantly reshape the distribution of wealth.

Finally, we stress that our results are not statements about total welfare and not an endorsement of a progressive capital tax. As discussed in the introduction, our econometric approach only generates empirical estimates of the distributional effects of taxes and other policies, it does not measure any distortions or costs associated with such policies. These estimates are intended only to add to our knowledge of the overall impact of such a policy.

5 Conclusion

The economic, political, and social impact of rising income and wealth inequality around the world is difficult to ignore. In response to these trends, a number of different policy reforms have been proposed (Stiglitz, 2013; Atkinson, 2015). One of the most controversial proposals involves imposing a progressive capital tax, a policy first suggested for Europe by Piketty (2014). In his best-selling book, Piketty (2014) discusses the impact of several different progressive capital taxes in response to rapidly rising top wealth shares around much of Europe. This proposal has generated substantial controversy largely because of the potentially significant distortions that could result from this policy. While some of the potential advantages of progressive capital taxes are discussed by Piketty (2014) and others, so far no studies have quantitatively analyzed the potential for progressive capital taxes to reduce wealth inequality. This paper addresses this question.

We use rank-based empirical methods to analyze the distributional effects of a progressive capital tax similar to that proposed by Piketty (2014). We focus on the U.S. and use the detailed wealth shares data of Saez and Zucman (2016) and other sources to estimate the econometric factors that shape the U.S. wealth distribution. In all scenarios we consider, we find that a progressive capital tax of 1-2% levied on just 1% of households substantially reshapes the distribution of wealth and reduces inequality. The exact impact of this capital tax depends on the current trajectory of U.S. inequality and on the extent to which households may successfully evade the tax or reduce savings in response to the tax.

The numerical estimates of the distributional effects of capital taxes we present in this paper are a useful first step. Perhaps our most important contribution, however, is to provide a rigorous and robust framework by which to analyze distributional effects of progressive capital taxes under any assumptions about the trajectory of future inequality and the impact of these taxes on household behavior and general equilibrium.

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Household Wealth Percent Rank	No Tax	Wealth Shares for Scenario 1	Wealth Shares for Scenario 2	Wealth Shares for Scenario 3
0-0.01	11.1%	1.5%	3.1%	0.9%
0.01-0.1	10.8%	4.0%	6.1%	3.0%
0.1-0.5	12.4%	8.4%	10.2%	7.1%
0.5-1	7.2%	6.7%	7.3%	6.2%
1-10	35.7%	44.9%	43.0%	45.2%
10-100	22.8%	34.6%	30.3%	37.7%

Table 1: Household wealth shares with a 1-2% progressive capital tax on the top 1% of households under Scenarios 1-3 for stationary parameter estimates. The baseline scenario, Scenario 1, assumes that a 1% capital tax reduces the growth rate of wealth of the taxed household by 1%, Scenario 2 assumes that a 1% capital tax reduces the growth rate of wealth of the taxed household by 0.5%, and Scenario 3 assumes that a 1% capital tax reduces the growth rate of wealth of the taxed household by 1.5%.

Household Wealth Percent Rank	No Tax	Wealth Shares for Scenario 1	Wealth Shares for Scenario 2	Wealth Shares for Scenario 3
0-0.01	48.4%	2.8%	9.1%	1.3%
0.01-0.1	24.7%	6.4%	12.6%	4.1%
0.1-0.5	9.1%	10.2%	13.1%	8.2%
0.5-1	2.7%	7.0%	7.0%	6.5%
1-10	9.5%	42.5%	35.0%	44.6%
10-100	5.6%	30.9%	23.2%	35.2%

Table 2: Household wealth shares with a 1-2% progressive capital tax on the top 1% of households under Scenarios 1-3 for non-stationary parameter estimates. The baseline scenario, Scenario 1, assumes that a 1% capital tax reduces the growth rate of wealth of the taxed household by 1%, Scenario 2 assumes that a 1% capital tax reduces the growth rate of wealth of the taxed household by 0.5%, and Scenario 3 assumes that a 1% capital tax reduces the growth rate of wealth of the taxed household by 1.5%.

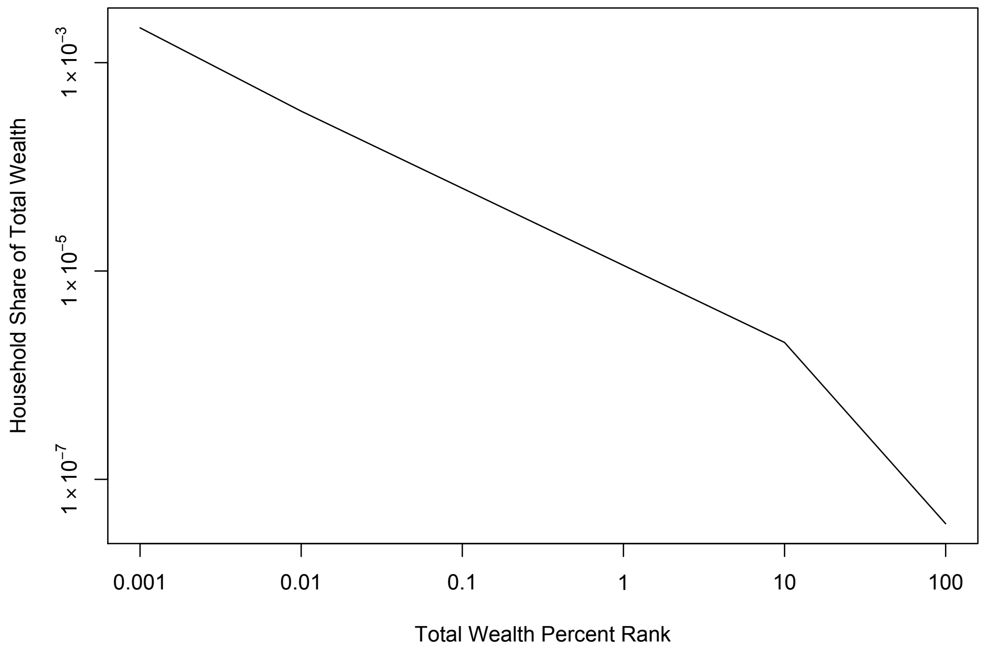

Figure 1: Household wealth shares matched to the 2012 U.S. wealth distribution.

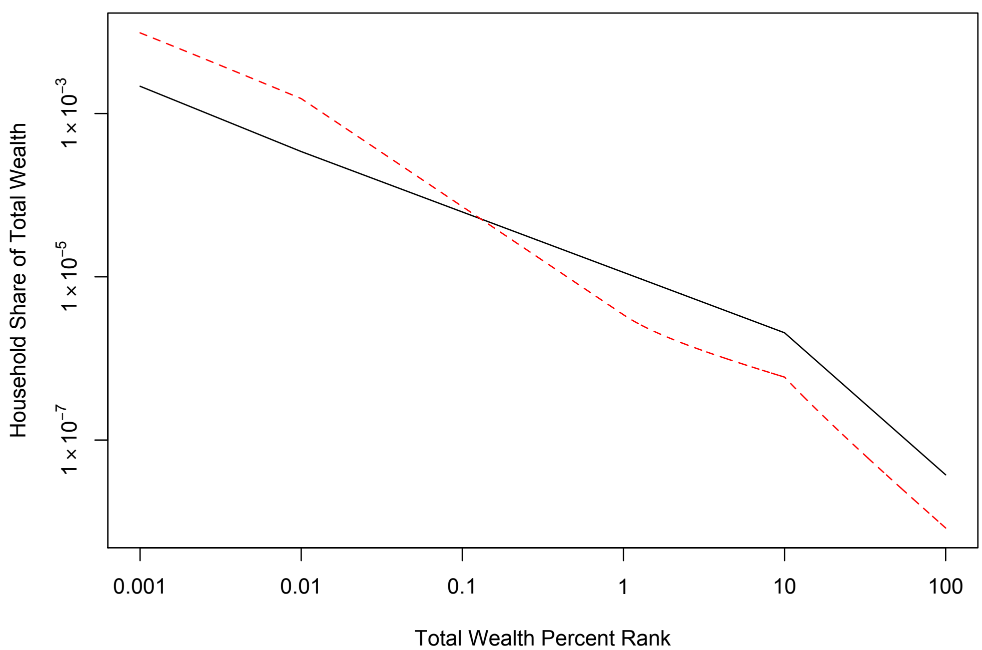

Figure 2: Household wealth shares for stationary (solid black line) and non-stationary (dashed red line) parameter estimates.

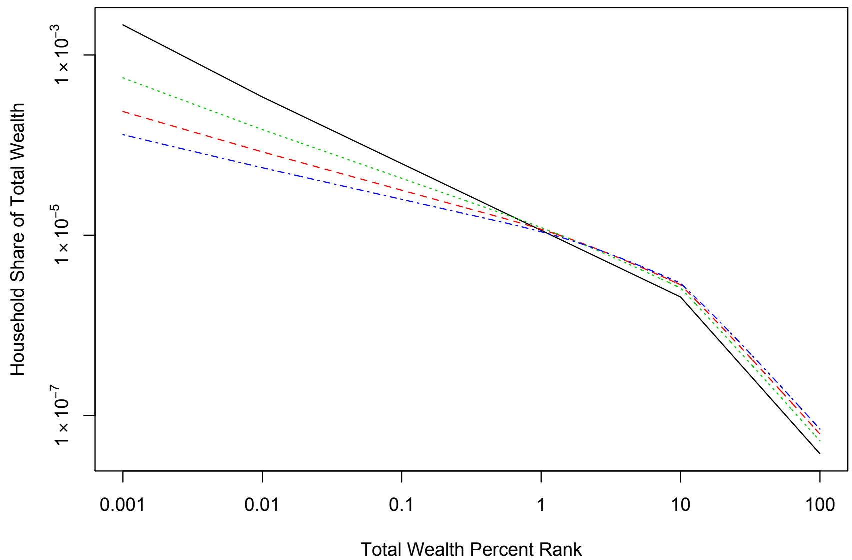

Figure 3: Household wealth shares with and without a 1-2% progressive capital tax on the top 1% of households under Scenarios 1-3 for stationary parameter estimates. The solid black line corresponds to no tax, the dashed red line corresponds to Scenario 1, the dotted green line corresponds to Scenario 2, and the dot-dashed blue line corresponds to Scenario 3.

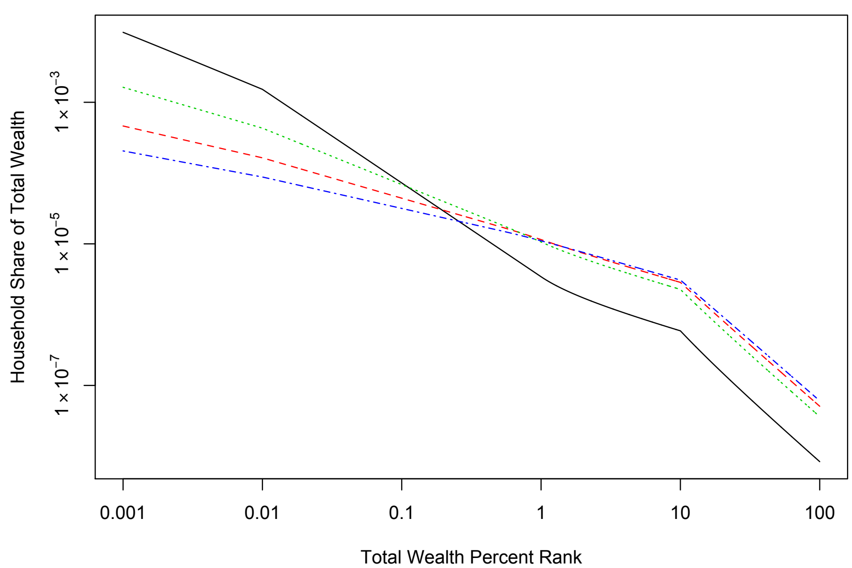

Figure 4: Household wealth shares with and without a 1-2% progressive capital tax on the top 1% of households under Scenarios 1-3 for non-stationary parameter estimates. The solid black line corresponds to no tax, the dashed red line corresponds to Scenario 1, the dotted green line corresponds to Scenario 2, and the dot-dashed blue line corresponds to Scenario 3.